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How to Prepare *for*

Quantitative Aptitude *for the* CAT

COMMON ADMISSION TEST

ARUN SHARMA

New Pattern

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BLOCK 1

CHAPTERS

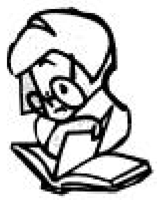
- Number Systems
- Progressions

INTRODUCTION

As already mentioned in the introductory note, Block 1 constituted the most crucial aspect of the Quantitative Aptitude Section in the paper & pen version of the CAT. Throughout the decade 1999 to 2008, almost 30–50% of the total questions in every CAT paper came from the two chapters given in this block. However, the online CAT has shifted this weightage around, and consequently, the importance of Block 1 has been reduced to around 20–25% of the total marks in the section.

Thus, although Block 1 remains an important block for your preparations, it has lost its pre-eminence (as reflected in the strategy—"Do Block 1 well and you can qualify the QA section"). However, this does not change the need for you to go through this block in great depth.

Thus our advice to you is: *Go through this block in depth and try to gain clarity of concepts as well as exposure to questions for honing your ability to do well in this area.*



...BACK TO SCHOOL

- **Chapters in this Block: Number Systems and Progressions**
- **Block Importance – 20–25%**

The importance of this block can be gauged from the table below:

Year	*% of Marks from Block I*	*Qualifying Score (approx score for 96 percentile)*
2000	48%	35%
2001	36%	35%
2002	36%	35%
2003 (cancelled)	30%	32%
2003 (retest)	34%	35%
2004	32%	35%
2005	40%	35%
2006	32%	40%
2007	24%	32%
2008	40%	35%
Online CAT 2009	15–30%	60% with no errors

As you can see from the table above, doing well in this block alone could give you a definite edge and take you a long way to qualifying the QA section. Although, the online CAT has significantly varied the relative importance to this block, the importance of this block remains high. Besides, there is a good chance that once the IIMs get their act together in the context of the online CAT and its question databases—the pre eminence of this block of chapters might return.

Hence, understanding the concepts involved in these chapters properly and strengthening your problem solving experience could go a long way towards a good score.

Before we move into the individual chapters of this block, let us first organize our thinking by looking at the core concepts that we had learnt in school with respect to these chapters.

PREASSESSMENT TEST

This test consists of 25 questions based on the chapters of BLOCK ONE (Number Systems and Progressions). Do your best in trying to solve each question.

The time limit to be followed for this test is 30 minutes. However, after the 30 minutes is over continue solving till you have spent enough time and paid sufficient attention to each question. After you finish thinking about each and every question of the test, check your scores. Then go through the SCORE INTERPRETATION ALGORITHM given at the end of the test to understand the way in which you need to approach the chapters inside this block.

1. The number of integers n satisfying $-n + 2 \geq 0$ and $2n \geq 4$ is
   (a) 0 (b) 1
   (c) 2 (d) 3
2. The sum of two integers is 10 and the sum of their reciprocals is 5/12. Then the larger of these integers is
   (a) 2 (b) 4
   (c) 6 (d) 8
3. If x is a positive integer such that $2x + 12$ is perfectly divisible by x, then the number of possible values of x is
   (a) 2 (b) 5
   (c) 6 (d) 12
4. Let K be a positive integer such that $k + 4$ is divisible by 7. Then the smallest positive integer n, greater than 2, such that $k + 2n$ is divisible by 7 equals.
   (a) 9 (b) 7
   (c) 5 (d) 3
5. $2^{73} - 2^{72} - 2^{71}$ is the same as
   (a) 2^{69} (b) 2^{70}
   (c) 2^{71} (d) 2^{72}
6. Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?
   (a) 15 (b) 9
   (c) 11 (d) 5
7. x, y and z are three positive integers such that $x > y > z$. Which of the following is closest to the product xyz?
   (a) $xy(z-1)$ (b) $(x-1)yz$
   (c) $(x-y)xy$ (d) $x(y+1)z$
8. A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let p be a prime number greater than 5, then $(p^2 - 1)$ is
   (a) never divisible by 6.
   (b) always divisible by 6, and may or may not be divisible by 12.
   (c) always divisible by 12, and may or may not be divisible by 24.
   (d) always divisible by 24.
9. Iqbal dealt some cards to Mushtaq and himself from a full pack of playing cards and laid the rest aside. Iqbal then said to Mushtaq "If you give me a certain number of your cards, I will have four times as many cards as you will have. If I give you the same number of cards, I will have thrice as many cards as you will have". Of the given choices, which could represent the number of cards with Iqbal?
   (a) 9 (b) 31
   (c) 12 (d) 35
10. In Sivkasi, each boy's quota of match sticks to fill into boxes is not more than 200 per session. If he reduces the number of sticks per box by 25, he can fill 3 more boxes with the total number of sticks assigned to him. Which of the following is the possible number of sticks assigned to each boy?
   (a) 200 (b) 150
   (c) 125 (d) 175
11. Alord got an order from a garment manufacturer for 480 Denim Shirts. He bought 12 sewing machines and appointed some expert tailors to do the job. However, many didn't report for duty. As a result, each of those who did, had to stitch 32 more shirts than originally planned by Alord, with equal distribution of work. How many tailors had been appointed earlier and how many had not reported for work?
   (a) 12, 4 (b) 10, 3
   (c) 10, 4 (d) None of these
12. How many 3-digit even numbers can you form such that if one of the digits is 5, the following digit must be 7?
   (a) 5 (b) 405
   (c) 365 (d) 495
13. To decide whether a number of n digits is divisible by 7, we can define a process by which its magnitude is reduced as follows: ($i_1, i_2, i_3.....i_n$ are the digits of the number, starting from the most significant digit).

$$i_1 i_2 \ldots i_n \Rightarrow i_1 . 3^{n-1} + i_2 . 3^{n-2} + \ldots + I_n 3^0.$$

e.g. $259 \Rightarrow 2.3^2 + 5.3^1 + 9.3^0 = 18 + 15 + 9 = 42$

Ultimately the resulting number will be seven after repeating the above process a certain number of times.

After how many such stages, does the number 203 reduce to 7?

(a) 2 (b) 3
(c) 4 (d) 1

14. A third standard teacher gave a simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

(a) 14, 22 (b) 13, 62
(c) 19, 33 (d) 42, 28

15. If 8 + 12 = 2, 7 + 14 = 3 then 10 + 18 =?

(a) 10 (b) 4
(c) 6 (d) 18

16. Find the minimum integral value of n such that the division $55n/124$ leaves no remainder.

(a) 124 (b) 123
(c) 31 (d) 62

17. What is the value of k for which the following system of equations has no solution:
$2x - 8y = 3$; and $kx + 4y = 10$.

(a) −2 (b) 1
(c) −1 (d) 2

18. A positive integer is said to be a prime if it is not divisible by any positive integer other than itself and one. Let p be a prime number strictly greater than 3. Then, when $p^2 + 17$ is divided by 12, the remainder is

(a) 6 (b) 1
(c) 0 (d) 8

19. A man sells chocolates that come in boxes. Either full boxes or half a box of chocolates can be bought from him. A customer comes and buys half the number of boxes the seller has plus half a box. A second customer comes and buys half the remaining number of boxes plus half a box. After this, the seller is left with no chocolates box. How many chocolates boxes did the seller have before the first customer came?

(a) 2 (b) 3
(c) 4 (d) 3.5

20. X and Y are playing a game. There are eleven 50 paise coins on the table and each player must pick up at least one coin but not more than five. The person picking up the last coin loses. X starts. How many should he pick up at the start to ensure a win no matter what strategy Y employs?

(a) 4 (b) 3
(c) 2 (d) 5

21. If $a < b$, which of the following is always true?

(a) $a < (a + b) / 2 < b$
(b) $a < ab/2 < b$
(c) $a < b^2 - a^2 < b$
(d) $a < ab < b$

22. The money order commission is calculated as follows. From Rs. X to be sent by money order, subtract 0.01 and divide by 10. Get the quotient and add 1 to it, if the result is Y, the money order commission is Rs. 0.5Y. If a person sends two money orders to Aurangabad and Bhatinda for Rs. 71 and Rs. 48 respectively, the total commission will be

(a) Rs. 7.00 (b) Rs. 6.50
(c) Rs. 6.00 (d) Rs. 7.50

23. The auto fare in Ahmedabad has the following formula based upon the meter reading. The meter reading is rounded up to the next higher multiple of 4. For instance, if the meter reading is 37 paise, it is rounded up to 40 paise. The resultant is multiplied by 12. The final result is rounded off to nearest multiple of 25 paise. If 53 paise is the meter reading what will be the actual fare?

(a) Rs. 6.75 (b) Rs. 6.50
(c) Rs. 6.25 (d) Rs. 7.50

24. Juhi and Bhagyashree were playing simple mathematical puzzles. Juhi wrote a two digit number and asked Bhayashree to guess it. Juhi also indicated that the number is exactly thrice the product of its digits. What was the number that Juhi wrote?

(a) 36 (b) 24
(c) 12 (d) 48

25. It is desired to extract the maximum power of 3 from 24!, where $n! = n.(n-1).(n-2) \ldots 3.2.1$. What will be the exponent of 3?

(a) 8 (b) 9
(c) 11 (d) 10

Answers (Block 1 Preassessment Test)

1. (b)	2. (c)	3. (c)	4. (a)	5. (c)
6. (a)	7. (b)	8. (d)	9. (b)	10. (b)
11. (c)	12. (c)	13. (a)	14. (b)	15. (a)
16. (a)	17. (c)	18. (a)	19. (b)	20. (b)
21. (a)	22. (b)	23. (a)	24. (b)	25. (d)

SOLUTIONS (PREASSESSMENT TEST)

1. The only value that will satisfy will be 2.
2. ¼ + 1/6 will give you 5/12.
3. The possible values are 1, 2, 3, 4, 6 and 12. (i.e. the factors of 12)
4. k will be a number of the form $7n + 3$. Hence, if you take the value of n as 9, $k + 2n$ will become $7n + 3 + 18 = 7n + 21$. This number will be divisible by 7. The numbers 3, 5 and 7 do not provide us with this solution.
5. $2^{73} - 2^{72} - 2^{71} = 2^{71}(2^2 - 2 - 1) = 2^{71}(1)$. Hence option (c) is correct.
6. Solve through options.
7. The closest value will be option (b), since the percentage change will be lowest when the largest number is reduced by one.
8. This is a property of prime numbers greater than 5.
9. He could have dealt a total of 40 cards, in which case Mushtaq would get 9 cards. On getting one card from Mushtaq, the ratio would become 4:1, while on giving away one card to Mushtaq, the ratio would become 3:1.
10. Looking at the options you realise that the correct answer should be a multiple of 25 and 50 both. The option that satisfies the condition of increasing the number of boxes by 3 is 150. (This is found through trial and error.)
11. Trial and error gives you option 3 as the correct answer.
12. Given that the number must have a 57 in it and should be even at the same time, the only numbers possible are 570, 572, 574, 576 and 578. Also, if there is no 5 in the number, you will get 360 more numbers.
13. 203 becomes → $2.3^2 + 0 + 3.3^0 = 21$ → $2.3^1 + 1.3^0 = 7$. Hence, clearly two steps are required.
14. Trial and error will give option (b) as the correct answer, since $13 \times 62 = 26 \times 31$
15. The solutions are defined as the sum of digits of the answer. Hence, 10 is correct.
16. There are no common factors between 55 and 124. Hence the answer should be 124.
17. At $k = -1$, the two equations become inconsistent with respect to each other and there will then be no solution to the system of equations.
18. Try with 5, 7, 11. In each case the remainder is 6.
19. Trial and error gives you answer 3 (b) Option.
20. Picking up 4 coins will ensure that he wins the game.
21. Option (a) is correct (since the average of any two numbers lies between the numbers.
22. 8/2 + 5/2 = 6.5.
23. The answer will be $56 \times 12 = 672$ → 675. Hence, Rs. 6.75.
24. The given condition is satisfied only for 24.
25. The answer will be given by 8 + 2 = 10.

(This logic is explained in the number systems Chapter)

SCORE INTERPRETATION ALGORITHM

Block 1 Preassessment Test

(Use a similar process for one to six blocks on the basis of your performance).

If You Scored: < 7: (In Unlimited Time)

Step One: Go through the block one **Back to School** Section carefully. Grasp each of the concepts explained in that part carefully. In fact I would recommend that you go back to your Mathematics school books (ICSE/ CBSE) Class 8, 9 and 10 if you feel you need it.

Step Two: Move into the first chapter of the block. Viz Number Systems. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number systems. While doing so do not think about the time requirement. Once you finish solving LOD 1, revise the questions and their solution processes.

Step Three: After finishing LOD 1 of number systems, move into Chapter 2 of this block, (Progressions) and repeat the process, viz: Chapter theory comprehensively followed by solving LOD 1 questions.

Step Four: Go to the first and second review tests given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the LOD 1 questions for both the chapters.

Step Five: Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions. Concentrate on understanding each and every question and its underlying concept.

Step Six: Go to the third to fifth review tests given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

Step Seven: Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 and LOD 2. Repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

If You Scored: 7–15 (In Unlimited Time)

Although you are better than the person following the instructions above, obviously there is a lot of scope for the development of your score. You will need to work both on your concepts as well as speed. Initially emphasize more on the concept development aspect of your preparations, then move your emphasis onto speed development. The following process is recommended for you:

Step One: Go through the **block one Back to School Section** carefully. Revise each of the concepts explained in that part. Going through your 8th, 9th and 10th standard books will be an optional exercise for you. It will be recommended in case you scored in single digits, while if your score is in two digits, I leave the choice to you.

Step Two: Move into the first chapter of the block. Viz Number Systems. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number Systems. Once you finish solving LOD 1, revise the questions and their solution processes.

Step Three: After finishing LOD 1 of number systems, move into Chapter 2 of this block, (Progressions) and repeat the process, viz: Chapter theory comprehensively followed by solving LOD 1 questions.

Step Four: Go to the first and second review tests given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your score.

Step Five: Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions.

Step Six: Go to the third to fifth review tests given at the end of the block and solve it. Again while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

In case the growth in your score is not significant, go back to the theory of both the chapters and re-solve LOD 1 and LOD 2 of both the chapters. While doing so concentrate more on the LOD 2 questions.

Step Seven: Move to LOD 3 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions.

If You Scored 15+ (In Unlimited Time)

Obviously you are much better than the first two categories of students. Hence unlike them, your focus should be on developing your speed by picking up the shorter processes explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you:

Step One: Quickly review the concepts given in the block one Back to School Section. Only go deeper into a concept in case you find it new. Going back to school level books is not required for you.

Step Two: Move into the first chapter of the block: Number Systems. Go through the theory explained there carefully. Concentrate specifically on clearly understanding the concepts which are new to you. Work out the short cuts and in fact try to expand your thinking by trying to think of alternative (and expanded) lines of questioning with respect to the concept you are studying.

Then move onto the LOD 1 exercises. Solve each and every question provided under LOD 1 of Number Systems. While doing so, try to think of variations that you can visualize in the same questions and how you would handle them.

Step Three: After finishing LOD 1 of number systems, move into Chapter 2 of this block, (Progressions) and repeat the process, viz: Chapter theory with emphasis on picking up things that you are unaware of, followed by solving LOD 1 questions and thinking about their possible variations.

Step Four: Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

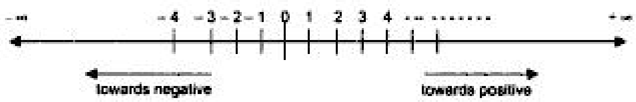
Step Six: Go to the first to fifth review tests—given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your score.

Step Seven: Move to LOD 3 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

CORE CONCEPTS

I. The concept of the number line is one of the most crucial concepts in Quantitative Aptitude.

The number line is a line that starts from zero and goes towards positive infinity when it moves to the right and towards negative infinity when it moves to the left.



The difference between the values of any two points on the number line also gives the distance between the points.

Thus, for example if we look at the distance between the points + 3 and − 2 it will be given by their difference. $3 - (-2) = 3 + 2 = 5$.

II. Types of numbers –

We will be looking at the types of numbers in details, again when we go into the chapter of number systems. Let us first work out in our minds the various types of numbers. While doing so do not fail to notice that most of these number types occur in pairs (i.e. the definition of one of them, defines the other automatically).

Natural Numbers and Whole Numbers: Natural numbers, also called counting numbers, are the first numbers we learnt. They are the number set 1, 2, 3, 4 .... ∞.

On the other hand, whole numbers is the set of natural numbers plus, the number zero.

Thus, 0, 1, 2, 3 ....∞ is the set of whole number.

Integers and Decimals: All numbers that do not have a decimal in them are called integers. Thus, −3, −17, + 4, + 13, + 1473, 0 etc are all integers.

Obviously, decimal numbers are number which have a decimal value attached to them. Thus, 1.3, 14.76, − 12.24 etc are all decimal numbers, since they have certain values after the decimal point.

Before we move ahead, let us pause a brief while, to further understand decimals. As you shall see, the concept of decimals is closely related to the concept of division and divisibility. Suppose, I have 4 pieces of bread which I want to divide equally between two people. It is easy for me to do this, since I can give two whole pieces to each of them.

However, if we alter the situation in such a way, that I now have 5 pieces of bread to distribute equally amongst 2 people. What do I do?

I give two whole pieces each, to each of them. The 5th piece has to be divided equally between the two. I can no longer do this, without in some way breaking the 5th piece into 2 parts. This is the elementary situation that gives rise to the need for decimals in mathematics.

Going back to the situation above, my only option is to divide the 5th piece into two equal parts (which in quants are called as halves).

This concept has huge implications for problem solving especially once you recognise that a half (i.e. a .5 in the decimal) only comes when you divide a whole into two parts.

Thus, in fact, all standards decimals emerge out of certain fixed divisors.

Hence, for example, the divisor 2, gives rise to the decimal ·5.

Similarly the divisor 3 gives rise to the decimals ·33333 and ·66666 etc.

Prime Numbers and Composite Numbers Amongst natural numbers, there are three broad divisions–

Unity It is representative of the number 1.

Prime Numbers These are numbers which have no divisors/ factors apart from 1 and itself.

Composite Numbers On the other hand, are numbers, which have at least one more divisor apart from 1 and itself.

Note: A brief word about factors/ division – A number X is said to divide Y (or is said to be a divisor or factor of Y). When the division of Y/X leaves no remainder.

All composite numbers have the property that they can be written as a product of their prime factors.

Thus, for instance, the number 40 can be represented as: $40 = 2 \times 2 \times 2 \times 5$ or $40 = 2^3 \times 5^1$

This form of writing is called as the **standard form** of the composite number.

The difference between Rational and Irrational numbers: This difference is one of the critical but unfortunately one of the less well understood differences in elementary Mathematics.

The definition of Rational numbers: Numbers which can be expressed in the form p/q where $q \neq 0$ are called rational numbers.

Obviously, numbers which cannot be represented in the form p/q are called as irrational numbers.

However, one of the less well understood issues in this regard is what does this mean?

The difference becomes clear when the values of decimals are examined in details:

Consider the following numbers

(1) 4.2,

(2) 4.333….,

(3) 4.1472576345…….

What is the difference between the decimal values of the three numbers above?

To put it simply, the first number has what can be described as a finite decimal value. Such numbers can be expressed in the form *p*/*q* easily. Since 4.2 can be first written as 42/10 and then converted to 21/5.

Similarly, number like 4.5732 can be represented as 45732/10000. Thus, numbers having a finite terminating decimal value are rational.

Now, let us consider the decimal value: 4.3333……..

Such decimal values will continue endlessly i.e. they have no end. Hence, they are called **infinite decimals** (or non-terminating decimals).

But, we can easily see that the number 4.333.... can be represented as 13/3. Hence, this number is also rational. In fact, all numbers which have infinite decimal values, but have any recurring form within them can be represented in the *p*/*q* form.

For example the value of the number: 1.14814814814…. is 93/81.

(What I mean to say is that whenever you have any recurring decimal number, even if the value of '*q*' might not be obvious, but it will always exist.)

Thus, we can conclude that all numbers whose decimal values are infinite (non-terminating) but which have a recurring pattern within them are rational numbers.

This leaves us with the third kind of decimal values, viz. **Infinite non-recurring decimal values**. These decimals neither have a recurring pattern, nor do they have an end—they go on endlessly. For such numbers it is not possible to find the value of a denominator '*q*' which can be used in order to represent them as *p*/*q*. Hence, such numbers are called as irrational numbers.

In day to day mathematics, we come across numbers like $\sqrt{3}$, $\sqrt{5}$, $3\sqrt{7}$, π, e, etc. which are irrational numbers since they do not have a *p*/*q* representation.

[**Note**: $\sqrt{3}$ can also be represented as $3^{1/2}$, just as $3\sqrt{7}$ can be represented as $7^{1/3}$.]

An important Tip:

Rational and Irrational numbers do not mix.: This means that in case you get a situation where an irrational number has appeared while solving a question, it will remain till the end of the solution. It can only be removed from the solution if it is multiplied or divided by the same irrational number.

Consider an example: The area of an equilateral triangle is given by the formula $(\sqrt{3}/4) \times a^2$ (where a is the side of the equilateral triangle). Since, $\sqrt{3}$ is an irrational number, it remains in the answer till the end. Hence, the area of an equilateral triangle will always have a $\sqrt{3}$ as part of the answer.

Before we move ahead we need to understand one final thing about recurring decimals.

As I have already mentioned, recurring decimals have the property of being able to be represented in the *p*/*q* form. The question that arises is—Is there any process to convert a recurring decimal into a proper fraction?

Yes, there is. In fact, in order to understand how this operates, you first need to understand that there are two kinds of recurring decimals. The process for converting an infinite recurring decimal into a fraction basically varies for both of these types. Let's look at these one by one.

Type 1–Pure recurring decimals: These are recurring decimals where the recurrence starts immediately after the decimal point.

For example :0.5555… $= 0.\overline{5}$

3.242424… $= 3.\overline{24}$

5.362362… $= 5.\overline{362}$

The process for converting these decimals to fractions can be illustrated as:

0.5555 = 5/9

3.242424 = 3 + (24/99)

5.362362 = 5 + (362/999)

A little bit of introspection will tell you that what we have done is nothing but to put down the recurring part of the decimal as it is and dividing it by a group of 9's. Also the number of 9's in this group equals the number of digits in the recurring part of the decimal.

Thus, in the second case, the fraction is derived by dividing 24 by 99. (24 being the recurring part of the decimal and 99 having 2 nines because the number of digits in 24 is 2.)

Similarly, 0.43576254357625…. $= \dfrac{4357625}{9999999}$

Type 2–Impure recurring decimals: Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal. The process to convert these into a fraction is also best illustrated by an example:

Consider the decimal 0.435424242

$= 0.435\overline{42}$

The fractional value of the same will be given by: (43542 – 435)/99000. This can be understood in two steps.

Step 1: Subtract the non-recurring initial part of the decimal (in this case, it is 435) from the number formed by writing down the starting digits of the decimal value upto the digit where the recurring decimals are written for the first time;

Expanding the meaning—

Note: For 0.435424242, subtract 435 from 43542

Step 2: The number thus obtained, has to be divided by a number formed as follows; Write down as many 9's as the number of digits in the recurring part of the decimal. (in this case, since the recurring part '42' has 2 digits, we write down 2 9's.) These nines have to be followed by as many zeroes as the number of digits in the non recurring part of the decimal value. (In this case, the non recurring part of the decimal value is '435'. Since, 435 has 3 digits, attach three zeroes to the two nines to get the number to divide the result of the first step.)

Hence divide 43542 – 435 by 99000 to get the fraction.

Similarly, for 3.436213213 we get $\frac{436213 - 436}{999000}$.

Let us now move onto our next topic –

Tables and their visualization: Imagine the number line and a frog sitting at point zero of the number line. Let us say that the frog always jumps an equal distance (say 2 units.)

Imagine that the frog sitting at the origin (point 0) of the number line starts jumping to its right through equidistant jumps of exactly 2 units. It will first land on the point represented by the number 2 on the number line. It's next jump will make it land on the number 4, then 6, then 8 and so on.

This is how you should visualise the table of any number.

Thus, a frog starting from 0 and jumping 7 units to the right will land on 7, then 14, 21, 28 and so forth. This frog's jumps represent the table of the number 7.

The meaning of 2n and 2n + 1: $2n$ means a number which is a multiple of the number 2. Since, this can be visualised as a frog starting from the origin and jumping 2 units to the right in every jump, you can also say that this frog represents $2n$.

(**Note**: Multiples of 2, are even numbers. Hence, $2n$ is also used to denote even numbers.)

So, what does $2n + 1$ mean?

Well, simply put, if you place the above frog on the point represented by the number 1 on the number line then the frog will reach points such as 3, 5, 7, 9, 11 .....and so on. This essentially means that the points the frog now reaches are displaced by 1 unit to the right of the $2n$ frog. In mathematical terms, this is represented as $2n + 1$.

In other words, $2n + 1$ also represents numbers which leave a remainder of 1, when divided by 2. (**Note**: This is also the definition of an odd number. Hence, in Mathematics ($2n + 1$) is used to denote an odd number. Also note that taken together $2n$ and $2n + 1$ denote the entire set of integers. i.e. all integers from $-\infty$ to $+\infty$ on the number line can be denoted by either $2n$ or $2n + 1$. This happens because when we divide any integer by 2, there are only two results possible with respect to the remainder obtained, viz: A remainder of zero ($2n$) or a remainder of one ($2n + 1$).

This concept can be expanded to represent integers with respect to any number. Thus, in terms of 3, we can only have three types of integers $3n$, $3n + 1$ or $3n + 2$ (depending on whether the integer leaves a remainder 0, 1 or 2 respectively when divided by 3.) Similarly, with respect to 4, we have 4 possibilities – $4n$, $4n + 1$, $4n + 2$ or $4n + 3$.

This form of representation of integers is extremely crucial in logically solving QA.

Rules of Indices: Indices means the power on a number. Many mathematical situations require us to be able to use the rules of indices. Hence, understanding these rules and their appropriate use might go a long way towards helping you in developing your skills in Quants.

The following rules apply for indices.

(1) $a^m \times a^n = a^{(m+n)}$. Thus, $2^3 \times 2^5 = 2^8$

(2) $a^m/a^n = a^{m-n}$. Thus $2^5/2^2 = 2^3$

(3) $a^m = 1/a^{-m}$ or $a^{-m} = 1/a^m$. Thus, $3^{-4} = 1/3^4$

(4) $(a^b)^c = a^{bc}$. Thus, $(5^2)^4 = 5^8$

(5) $a^0 = 1$ for all values of a. Thus, $7^0 = 1$

Besides, the following principles apply for indices:

(1) If $a^m = n$, then $a = n^{1/m}$

(2) $a^n/b^n = (a/b)^n$ or vice versa

(3) $a^{b^c} \neq a^{bc}$. Thus $2^{3^4} = 2^{81}$ and not 2^{12}.

Squares and square roots When any number is multiplied by itself, it is called as the square of the number.

Thus, $3 \times 3 = 3^2 = 9$

Squares have a very important role to play in mathematics. In the context of preparing for CAT and other Management exams, it might be a good idea to be able to recollect the squares of 2 digit numbers.

Let us now go through the following Table 1.1 carefully:

Table 1.1

Number	*Square*	*Number*	*Square*	*Number*	*Square*
1	1	35	1225	69	4761
2	4	36	1296	70	4900
3	9	37	1369	71	5041
4	16	38	1444	72	5184
5	25	39	1521	73	5329
6	36	40	1600	74	5476
7	49	41	1681	75	5625
8	64	42	1764	76	5776
9	81	43	1849	77	5929
10	100	44	1936	78	6084
11	121	45	2025	79	6241
12	144	46	2116	80	6400
13	169	47	2209	81	6561
14	196	48	2304	82	6724
15	225	49	2401	83	6889
16	256	50	2500	84	7056
17	289	51	2601	85	7225
18	324	52	2704	86	7396
19	361	53	2809	87	7561
20	400	54	2916	88	7744
21	441	55	3025	89	7921
22	484	56	3136	90	8100
23	529	57	3249	91	8281
24	576	58	3364	92	8464
25	625	59	3481	93	8649
26	676	60	3600	94	8836
27	729	61	3721	95	9025
28	784	62	3844	96	9216
29	841	63	3969	97	9409
30	900	64	4096	98	9604
31	961	65	4225	99	9801
32	1024	66	4356	100	10000
33	1089	67	4489		
34	1156	68	4624		

So, how does one get these numbers onto one's finger tips? Does one memorize these values or is there a simpler way?

Yes indeed! There is a very convenient process when it comes to memorising the squares of the first 100 numbers.

First of all, you are expected to memorise the squares of the first 30 numbers. In my experience, I have normally seen that most students already know this. The problem arises with numbers after 30. You do not need to worry about that.

Just follow the following processes and you'll know all squares upto 100.

Trick 1: For squares from 51 to 80 – (**Note**: This method depends on your memory of the first thirty squares.)

The process is best explained through an example.

Suppose, you have to get an answer for the value of 67^2. Look at 67 as (50 + 17). The 4 digit answer will have two parts as follows:

__ __ (First 2 digits) __ __ (Last 2 digits)

The last two digits will be the same as the last two digits of the square of the number 17. (The value 17 is derived by looking at the difference of 67 with respect to 50.)

Since, $17^2 = 289$, you can say that the last two digits of 67^2 will be 89. (i.e. the last 2 digits of 289.) Also, you will need to carry over the '2' in the hundreds place of 289 to the first part of the number.

The first two digits of the answer will be got by adding 17 (which is got from 67 – 50) and adding the carry over (2 in this case) to the number 25. (Standard number to be used in all cases.) Hence, the first two digits of the answer will be given by 25 + 17 + 2 = 44.

Hence, the answer is $67^2 = 4489$.

Similarly, suppose you have to find 76^2.

Step 1: 76 = 50 + 26.

Step 2: 26^2 is 676. Hence, the last 2 digits of the answer will be 76 and we will carry over 6.

Step 3: The first two digits of the answer will be 25 + 26 + 6 = 57.

Hence, the answer is 5776.

This technique will take care of squares from 51 to 80 (if you remember the squares from 1 to 30). You are advised to use this process and see the answers for yourself.

Squares for Numbers from 31 to 50 Such numbers can be treated in the form $(50 - x)$ and the above process modified to get the values of squares from 31 to 50. Again, to explain we will use an example. Suppose you have to find the square of 41.

Step 1: Look at 41 as (50 – 9).

Again, similar to what we did above, realise that the answer has two parts—the first two and the last two digits.

Step 2: The last two digits are got by the last two digits in the value of $(-9)^2 = 81$. Hence, 81 will represent the last two digits of 41^2.

Step 3: The first two digits are derived by $25 - 9 = 16$ (where 25 is a standard number to be used in all cases and -9 comes from the fact that $(50 - 9) = 41$).

Hence, the answer is 1681.

Note: In case there had been a carry over from the last two digits it would have been added to 16 to get the answer.

For example, in finding the value of 36^2 we look at $36 = (50 - 14)$

Now, $(-14)^2 = 196$. Hence, the last 2 digits of the answer will be 96. The number '1' in the hundreds place will have to be carried over to the first 2 digits of the answer.

The, first two digits will be $25 - 14 + 1 = 12$

Hence, $36^2 = 1296$.

With this process, you are equipped to find the squares of numbers from 31 to 50.

Finding squares of numbers between 81 to 100:

Suppose you have to find the value of 82^2. The following process will give you the answers.

Step 1: Look at 82 as $(100 - 18)$. The answer will have 4 digits whose values will be got by focusing on getting the value of the last two digits and that of the first two digits.

Step 2: The value of the last two digits will be equal to the last two digits of $(-18)^2$.

Since, $(-18)^2 = 324$, the last two digits of 82^2 will be 24. The '3' in the hundreds place of $(-18)^2$ will be carried over to the other part of the answer (i.e. the first two digits).

Step 3: The first two digits will be got by: $82 + (-18) + 3$ Where 82 is the original number; (-18) is the number obtained by looking at 82 as $(100 - x)$; and 3 is the carry over from $(-18)^2$.

Similarly, 87^2 will give you the following thought process: $87 = 100 - 13 \rightarrow (-13)^2 = 169$. Hence, 69 are the last two digits of the answer $\rightarrow$ Carry over 1. Consequently, $87 + (-13) + 1 = 75$ will be the first 2 digits of the answer.

Hence, $87^2 = 7569$.

With these three processes you will be able to derive the square of any number up to 100.

Properties of squares:

1. When a perfect square is written as a product of its prime factors each prime factor will appear an even number of times.
2. The difference between the squares of two consecutive natural numbers is always equal to the sum of the natural numbers. Thus, $41^2 - 40^2 = (40 + 41) = 81$.

   This property is very useful when used in the opposite direction—i.e. Given that the difference between the squares of two consecutive integers is 81, you should immediately realise that the numbers should be 40 and 41.
3. The square of a number ending in 1, 5 or 6 also ends in 1, 5 or 6 respectively.
4. The square of any number ending in 5: The last two digits will always be 25. The digits before that in the answer will be got by multiplying the digits leading up to the digit 5 in the number by 1 more than itself.

   Illustration:

   $$85^2 = \_\_25.$$

   The missing digits in the above answer will be got by $8 \times (8 + 1) = 8 \times 9 = 72$. Hence, the square of 85 is given by 7225.

   Similarly, $135^2 = \_\_25$. The missing digits are $13 \times 14 = 182$. Hence, $135^2 = 18225$.
5. The value of a perfect square has to end in 1, 4, 5, 6, 9 or an even number of zeros. In other words, a perfect square cannot end in 2, 3, 7, or 8 or an odd number of zeros.
6. If the units digit of the square of a number is 1, then the number should end in 1 or 9.
7. If the units digit of the square of a number is 4, then the units digit of the number is 2 or 8.
8. If the units digit of the square of a number is 9, then the units digit of the number is 3 or 7.
9. If the units of the square of a number is 6, then the unit's digit of the number is 4 or 6.
10. The sum of the squares of the first 'n' natural numbers is given by

    $$[(n)(n + 1)(2n + 1)]/6.$$
11. The square of a number is always non-negative.
12. Normally, by squaring any number we increase the value of the number. The only integers for which this is not true are 0 and 1. (In these cases squaring the number has no effect on the value of the number).

    Further, for values between 0 to 1, squaring the number reduces the value of the number. For example $0.5^2 < 0.5$.

Finding the Square Root of a Given Number

Say 7016

Step 1: Write down the number 7016 as a product of its

Prime factors. $7016 = 2 \times 2 \times 2 \times 2 \times 21 \times 21$

$= 2^4 \times 21^2$

Step 2: The required square root is obtained by halving the values of the powers.

Hence, $\sqrt{7056} = 2^2 \times 21^1$

CUBES AND CUBE ROOTS

When a number is multiplied with itself two times, we get the cube of the number.

Thus, $x \times x \times x = x^3$

Method to find out the cubes of 2 digit numbers: The answer has to consist of 4 parts, each of which has to be calculated separately.

The first part of the answer will be given by the cube of the ten's digit.

Suppose you have to find the cube of 28.

The first step is to find the cube of 2 and write it down.

$2^3 = 8.$

The next three parts of the number will be derived as follows. Derive the values 32, 128 and 512.

(by creating a G. P. of 4 terms with the first term in this case as 8, and a common ratio got by calculating the ratio of the unit's digit of the number with its tens digit. In this case the ratio is 8/2 = 4.)

Now, write the 4 terms in a straight line as below. Also, to the middle two terms add double the value.

	8	32	128	512
+		64	256	
	21	9	5	2
				(carry over 51)
	(8 + 13)	(32 + 64 + 43 = 139) Carry over 13	(128 + 256 + 51 = 435) (Carry over 43)	

Hence, $28^3 = 21952$

Properties of Cubes

1. When a perfect cube is written in its standard form the values of the powers on each prime factor will be a multiple of 3.
2. In order to find the cube root of a number, first write it in its standard form and then divide all powers by 3.

   Thus, the cube root of $3^6 \times 5^9 \times 17^3 \times 2^6$ is given by $3^2 \times 5^3 \times 17 \times 2^2$
3. The cubes of all numbers (integers and decimals) greater than 1 are greater than the number itself.
4. $0^3 = 0$, $1^3 = 1$ and $-1^3 = -1$. These are the only three instances where the cube of the number is equal to the number itself.
5. The value of the cubes of a number between 0 and 1 is lower than the number itself. Thus, $0.5^3 < 0.5^2 < 0.5$.
6. The cube of a number between 0 and – 1 is greater than the number itself. $(-0.2)^3 > -0.2$.
7. The cube of any number less than –1, is always lower than the number. Thus, $(-1.5^3 < (-1.5)$.

The BODMAS Rule: It is used for the ordering of mathematical operations in a mathematical situation:

In any mathematical situation, the first thing to be considered is Brackets followed by Division, Multiplication, Addition and Subtraction in that order.

Thus $3 \times 5 - 2 = 15 - 2 = 13$

Also, $3 \times 5 - 6 \div 3 = 15 - 2 = 13$

Also, $3 \times (5 - 6) \div 3 = 3 \times (-1) \div 3 = -1.$

Operations on Odd and Even numbers

ODDS

Odd × odd	= Odd
Odd + odd	= Even
Odd – odd	= Even
Odd ÷ odd	= odd

EVENS

Even × Even = Even
Even + Even = Even
Even – even = Even
Even ÷ even = Even or odd

ODDS & EVENS

Odd × Even	= Even
Odd + Even	= Odd
Odd – Even	= Odd
Even ÷ odd	= Even

Odd ÷ Even → Not divisible

SERIES OF NUMBERS

In many instances in Mathematics we are presented with a series of numbers formed simply when a group of numbers is written together. The following are examples of series:

1. 3, 5, 8, 12, 17...
2. 3, 7, 11, 15, 19...(Such series where the next term is derived by adding a certain fixed value to the previous number are called as Arithmetic Progressions).

3. 5, 10, 20, 40 .....(Such series where the next term is derived by multiplying the previous term by a fixed value are called as Geometric Progressions).

   (**Note**: You will study AP and GP is details in the chapter of progressions which is chapter 2 of this block.)
4. 2, 7, 22, 67 ....
5. 1/3, 1/5, 1/7, 1/9, 1/11...
6. $1/1^2$, $1/2^2$, $1/3^2$, $¼^2$, $1/5^2$...
7. $1/1^3$, $1/3^3$, $1/5^3$...

Remember the following points at this stage:

1. AP and GP are two specific instances of series. They are studied in details only because they have many applications and have defined rules.
2. Based on the behaviour of their sums, series can be classified as:

Divergent: These are series whose sum to '*n*' terms keeps increasing and reaches infinity for infinite terms.

Convergent: Convergent series have the property that their sum tends to approach an upper limit/lower limit as you include more terms in the series. They have the additional property that even when infinite terms of the series are included they will only reach that value and not cross it.

For example consider the series;

$1/1^2 + 1/3^2 + 1/5^2 + 1/7^2$ ...

It is evident that subsequent terms of this series keep getting smaller. Hence, their value becomes negligible after a few terms of the series are taken into account.

If taken to infinite terms, the sum of this series will reach a value which it will never cross. Such series are called convergent, because their sum converges to a limit and only reaches that limit for infinite terms.

Note: Questions on finding infinite sums of convergent series are very commonly asked in most aptitude exams including CAT and XAT.

NOTE FOR THE READER: NOW THAT YOU ARE THROUGH WITH THE BACK TO SCHOOL SECTION, YOU ARE READY TO PROCEED INTO THE CHAPTERS OF THIS BLOCK. HAPPY SOLVING!!

1

NUMBER SYSTEMS

INTRODUCTION

The chapter of number systems is amongst the most important chapters in the whole of mathematics syllabus for the CAT examination. Students are advised to go through this chapter with utmost care understanding each and every question type on this topic. The CAT has consistently set between 10–15 marks based on the concepts of this chapter. Hence, going through this chapter and its concepts properly is very imperative for you. Seen in the context of the qualifying score in the CAT being in the range of 12–14 marks (for Q A), this leaves us with a fair idea of the critical importance of this chapter. It would be a good idea to first go through the basic definitions of all types of numbers (something I have found to be surprisingly very less known about). The student is also advised to go through the solutions of the various questions illustrated in this chapter. Besides, while solving this chapter, try to maximise your learning experience with every problem that you solve. To start off, the following pictorial representation of the types of numbers will help you improve year quality of comprehension of different types of numbers.

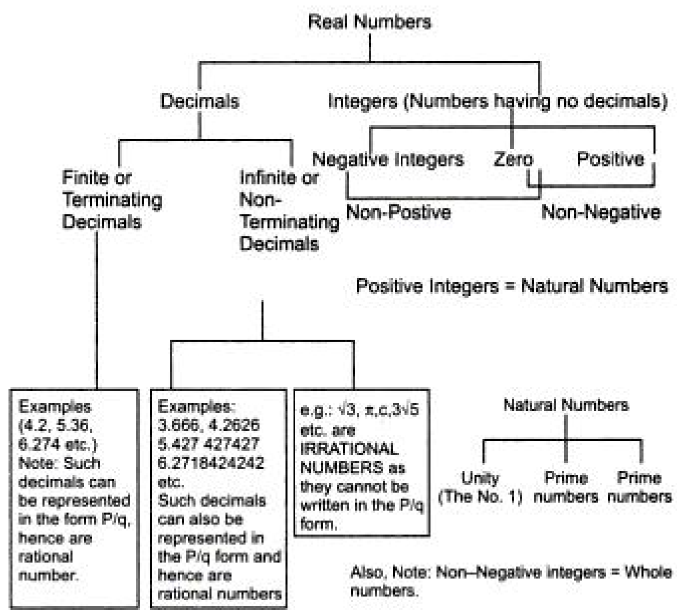
DEFINITIONS

Natural Numbers These are the numbers (1, 2, 3 etc.) that are used for counting. In other words, all positive integers are natural numbers.

There are infinite natural numbers and the number 1 is the least natural number.

Examples of natural numbers: 1, 2, 4, 8, 32, 23, 4321 and so on.

The following numbers are examples of numbers that are not natural: –2, –31, 2.38, 0 and so on.



Based on divisibility, there could be two types of natural numbers: *Prime and Composite*.

Prime Numbers A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

***Note*:** Unity (i.e. 1) is not a prime number.

Some Properties of Prime Numbers

- The lowest prime number is 2.
- 2 is also the only even prime number.
- The lowest odd prime number is 3.

- The remainder when a prime number $p \geq 5$ is divided by 6 is 1 or 5. However, if a number on being divided by 6 gives a remainder of 1 or 5 the number need not be prime.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 24 is 1.
- For prime numbers $p > 3$, $p^2 - 1$ is divisible by 24.
- Prime Numbers between 1 to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
- Prime Numbers between 100 to 200 are: 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.
- If a and b are any two odd primes then $a^2 - b^2$ is composite. Also, $a^2 + b^2$ is composite.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 12 is 1.

SHORT CUT PROCESS

To Check Whether a Number is Prime or Not

To check whether a number N is prime, adopt the following process.

(a) Take the square root of the number.

(b) Round of the square root to the immediately lower integer. Call this number z. For example if you have to check for 181, its square root will be 13.... Hence, the value of z, is this case will be 13.

(c) Check for divisibility of the number N by all prime numbers below z. If there is no prime number below the value of z which divides N then the number N will be prime.

To illustrate :-

The value of $\sqrt{239}$ lies between 15 to 16. Hence, take the value of z as 16.

Prime numbers less than 16 are 2, 3, 5, 7, 11 and 13, 239 is not divisible by any of these. Hence you can conclude that 239 is a prime number.

A Brief Look into Why this Works?

Suppose you are asked to find the factors of the number 40.

An untrained mind will find the factors as : 1, 2, 4, 5, 8, 10, 20 and 40.

The same task will be performed by a trained mind as follows:

	1	×	40
	2	×	20
	4	×	10
and	5	×	8

i.e., The discovery of one factor will automatically yield the other factor. In other words, factors will appear in terms of what can be called as factor pairs. The locating of one factor, will automatically pinpoint the other one for you. Thus, in the example above, when you find 5 as a factor of 40, you will automatically get 8 too as a factor.

Now take a look again at the pairs in the example above. If you compare the values in each pair with the square root of 40 (i.e. 6.___) you will find that for each pair the number in the left column is lower than the square root of 40, while the number in the right column is higher than the square root of 40.

This is a property for all numbers and is always true.

Hence, we can now phrase this as: Whenever, you have to find the factors of any number N, you will get the factors in pairs (i.e. factor pairs). Further, the factor pairs will be such that in each pair of factors, one of the factors will be lower than the square root of N while the other will be higher than the square root of N.

As a result of this fact one need not make any effort to find the factors of a number above the square root of the number. These come automatically. All you need to do is to find the factors below the square root of the number.

Extending this logic, we can say that if we are not able to find a factor of a number upto the value of its square root, we will not be able to find any factor above the square root and the number under consideration will be a prime number. This is the reason why when we need to check whether a number is prime, we have to check for factors only below the square root.

But, we have said that you need to check for divisibility only with the prime numbers below (and including) the square root of the number. What logic will explain this:

Let us look at an example to understand why you need to look only at prime numbers below the square root.

Uptil now, we have deduced that in order to check whether a number is prime, we just need to do a factor search below (and including) the square root.

Thus, for example, in order to find whether 181 is a prime number, we need to check with the numbers = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.

The first thing you will realise, when you first look at the list above is that all even numbers will get eliminated automatically (since no even number can divide an odd number and of course you will check a number for being prime only if it is odd!)

This will leave you with the numbers 3, 5, 7, 9, 11 and 13 to check 181.

Why do we not need to check with composite numbers below the square root? This will again be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked N for divisibility with 3 you would get either of two cases:

Case I: If N is divisible by 3: In such a case, N will automatically become non-prime and we can stop our checking. Hence, you will not need to check for the divisibility of the number by 9.

Case II: N is not divisible by 3: If N is not divisible by 3, it is obvious that it will not be divisible by 9. Hence, you will not need to check for the divisibility of the number by 9.

Thus, in either case, checking for divisibility by a composite number (9 in this case) will become useless. This will be true for all composite numbers.

Hence, when we have to check whether a number N is prime or not, we need to only check for its divisibility by prime factors below the square root of N.

Integers A set which consists of natural numbers, negative integers ($-1, -2, -3 \ldots -n \ldots$) and zero is known as the set of integers. The numbers belonging to this set are known as integers.

Composite Numbers It is a natural number that has at least one divisor different from unity and itself.

Every composite number n can be factored into its prime factors. (This is sometimes called the canonical form of a number.)

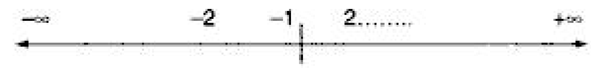
In mathematical terms: $n = p_1^m \cdot p_2^n \ldots p_k^s$, where $p_1, p_2 \ldots p_k$ are prime numbers called factors and $m, n \ldots k$ are natural numbers.

Thus, $24 = 2^3 \cdot 3, 84 = 7 \cdot 3 \cdot 2^2$ etc.

This representation of a composite number is known as the standard form of a composite number. It is an extermely useful form of seeing a composite number as we shall see.

Whole Numbers The set of numbers that includes all natural numbers and the number zero are called whole numbers. Whole number are also called as Non-negative integers.

The Concept of the Number Line The number line is a straight line between negative infinity on the left to infinity to the right.



The distance between any two points on the number line is got by subtracting the lower value from the higher value. Alternately, we can also start with the lower number and find the required addition to reach the higher number.

For example: The distance between the points 7 and –4 will be $7 - (-4) = 11$.

Real Numbers All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

The followning operations of addition, subtraction, multiplication and division are valid for both whole numbers and real numbers: [For any real or whole numbers a, b and c].

(a) Commutative property of addition: $a + b = b + a$.
(b) Associative property of addition: $(a + b) + c = a + (b + c)$.
(c) Commutative property of multiplication: $a \cdot b = b \cdot a$.
(d) Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
(e) Distributive property of multiplication with respect to addition: $(a + b)\ c = ac + bc$.
(f) Subtraction and division are defined as the inverse operations to addition and multiplication respectively.

Thus if $a + b = c$, then $c - b = a$ and if $q = a/b$ then $b \cdot q = a$ (where $b \neq 0$).

Division by zero is not possible since there is no number q for which $b \cdot q$ equals a non zero number a.

Rational Numbers A rational number is defined as number of the form a/b where a and b are integers and $b \neq 0$.

The set of rational numbers encloses the set of integers and fractions. The rules given above for addition, subtraction, multiplication and division also apply on rational numbers.

Rational numbers that are not integral will have decimal values. These values can be of two types:

(a) **Terminating (or finite) decimal fractions:** For example, $17/4 = 4.25$, $21/5 = 4.2$ and so forth.

(b) **Non-terminating decimal fractions:** Amongst non-terminating decimal fractions there are two types of decimal values:

(i) *Non-terminating periodic fractions*: These are non-terminating decimal fractions of the type $x \cdot a_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_n$. For example $\frac{16}{3}$ = 5.3333, 15.23232323, 14.287628762 876 … and so on.

(ii) *Non-terminating non-periodic fractions*: These are of the form $x \cdot b_1b_2b_3b_4 \ldots b_nc_1c_2c_3 \ldots c_n$. For example: 5.2731687143725186.…

Of the above categories, terminating decimal and non-terminating periodic decimal fractions belong to the set of rational numbers.

Irrational Numbers Fractions, that are non-terminating, non-periodic fractions, are irrational numbers.

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$ etc. In other words, all square and cube roots of natural numbers that are not squares and cubes of natural numbers are irrational. Other irrational numbers include π, e and so on.

Every positive irrational number has a negative irrational number corresponding to it.

All operations of addition, subtraction, multiplication and division applicable to rational numbers are also applicable to irrational numbers.

As briefly stated in the Back to school section, whenever an expression contains a rational and an irrational number together, the two have to be carried together till the end. In other words, an irrational number once it appears in the solution of a question will continue to appear till the end of the question. This concept is particularly useful in Geometry For example: If you are asked to find the ratio of the area of a circle to that of an equilateral triangle, you can expect to see a $\alpha/\sqrt{3}$ in the answer. This is because the area of a circle will always have a p component in it, while that of an equilateral triangle will always have $\sqrt{3}$.

You should realise that once an irrational number appears in the solution of a question, it can only disappear if it is multiplied or divided by the same irrational number.

THE CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural numbers n, and n_2.

If the numbers n_1 and n_2 are exactly divisible by the same number x, then x is a common divisor of n_1 and n_2.

The highest of all the common divisors of n_1 and n_2 is called as the GCD or the HCF. This is denoted as GCD (n_1, n_2).

Rules for Finding the GCD of Two Numbers n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all prime factors that are common to the standard forms of the numbers n_1 and n_2.

(c) Raise each of the common prime factors listed above to the lesser of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of the results of the previous step will be the GCD of n_1 and n_2.

Illustration: Find the GCD of 150, 210, 375.

Step 1: Writting down the standard form of numbers

$$150 = 5 \times 5 \times 3 \times 2$$
$$210 = 5 \times 2 \times 7 \times 3$$
$$375 = 5 \times 5 \times 5 \times 3$$

Step 2: Writing Prime factors common to all the three numbers is $5^1 \times 3^1$

Step 3: This will give the same result, i.e. $5^1 \times 3^1$

Step 4: Hence, the HCF will be $5 \times 3 = 15$

For practice, find the HCF of the following:

(a) 78, 39, 195
(b) 440, 140, 390
(c) 198, 121, 1331

THE CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let n_1, and n_2 be two natural numbers distinct from each other. The smallest natural number n that is exactly divisible by n_1 and n_2 is called the Least Common Multiple (LCM) of n_1 and n_2 and is designated as LCM (n_1, n_2).

Rule for Finding the LCM of 2 numbers n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all the prime factors, which are contained in the standard forms of either of the numbers.

(c) Raise each of the prime factors listed above to the highest of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of results of the previous step will be the LCM of n_1 and n_2.

Illustration: Find the LCM of 150, 210, 375.

Step 1: Writing down the standard form of numbers

$$150 = 5 \times 5 \times 3 \times 2$$
$$210 = 5 \times 2 \times 7 \times 3$$
$$375 = 5 \times 5 \times 5 \times 3$$

Step 2: Write down all the prime factors: that appear at least once in any of the numbers: 5, 3, 2, 7.

Step 3: Raise each of the prime factors to their highest available power (considering each to the numbers).

The LCM $= 2^1 \times 3^1 \times 5^3 \times 7^1 = 5250$.

Important Rule:

GCD (n_1, n_2). LCM $(n_1, n_2) = n_1 \cdot n_2$
i.e. The product of the HCF and the LCM equals the product of the numbers.

RULE FOR FINDING OUT HCF AND LCM OF FRACTIONS:

(A) HCF of two or more fractions is given by:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

(B) LCM of two or more fractions is given by:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Rules for HCF: If the HCF of x and y is G, then the HCF of

(i) x, $(x + y)$ is also G

(ii) x, $(x - y)$ is also G

(iii) $(x + y)$, $(x - y)$ is also G

HCF and LCM

Practice Exercise

(Typical questions asked in Exams)

1. Find the common factors for the numbers.
   (a) 24 and 64 (b) 42, 294 and 882
   (c) 60, 120 and 220
2. Find the HCF of
   (a) 420 and 1782 (b) 36 and 48
   (c) 54, 72, 198 (d) 62, 186 and 279
3. Find the LCM of
   (a) 13, 23 and 48 (b) 24, 36, 44 and 62
   (c) 22, 33, 45, and 72 (d) 13, 17, 21 and 33
4. Find the series of common multiples of
   (a) 54 and 36 (b) 33, 45 and 60

   [*Hint*: Find the LCM and then create an Arithmetic progression with the first term as the LCM and the common difference also as the LCM.]
5. The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is: [MAT]
   (a) 42 (b) 52
   (c) 62 (d) None of these

   Answer: (b) use HCF × LCM = product of numbers.
6. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time? [MAT]
   (a) 12:10 P.M. (b) 12:12 P.M.
   (c) 12:11 P.M. (d) None of these

   Answer: (d). The LCM of 50 and 48 being 1200, the two clocks will ring again after 1200 seconds.
7. 4 Bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours? [IIFT, MAT]
   (a) 3 (b) 4
   (c) 5 (d) 6

   Answer: (c). The LCM of 7, 8, 11 and 12 is 1848. Hence, the answer will be got by the greatest integer function of the ratio (10800)/(1848)=> 5.
8. On Ashok Marg three consecutive traffic lights change after 36, 42 and 72 seconds respectively. If the lights are first switched on at 9:00 A.M. sharp, at what time will they change simultaneously?
   (a) 9 : 08 : 04 (b) 9 : 08 : 24
   (c) 9 : 08 : 44 (d) None of these

   Answer (b). The LCM of 36,42 and 72 is 504. Hence, the lights will change simultaneously after 8 minutes and 24 seconds.
9. The HCF of 2472, 1284 and a third number 'N' is 12. If their LCM is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$, then the number '$N$' is:
   (a) $2^2 \times 3^2 \times 7^1$ (b) $2^2 \times 3^3 \times 103$

(c) $2^2 \times 3^2 \times 5^1$ (d) None of these

Answer: (c)

10. Two equilateral triangles have the sides of lengths 34 and 85 respectively.

    (a) The greatest length of tape that can measure both of them exactly is:

    Answer: HCF of 34 and 85 is 17.

    (b) How many such equal parts can be measured?

    Answer: $34/17 + 85/17 = 2 + 5 = 7$

11. Two numbers are in the ratio 17:13. If their HCF is 15, what are the numbers?

    Answer: 17×15 and 13×15 i.e. 255 and 195 respectively.

12. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The number of rows (minimum) that are required are:

    (a) 2 (b) 3
    (c) 10 (d) 11

    Answer: (c) 44/22 + 66/22 + 110/22 (Since 22 is the HCF)

13. Three runners running around a circular track, can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?

    (a) 22 (b) 33
    (c) 11 (d) 44

    (The answer will be the LCM of 2, 4 and 11/2. This will give you 44 as the answer).

14. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. The other number is?

    (a) 66 (b) 132
    (c) 198 (d) 99

    (Answer: $33 \times 264 = 66 \times n$. Hence, $n = 132$)

15. The greatest number which will divide: 4003, 4126 and 4249:

    (a) 43 (b) 41
    (c) 45 (d) None of these

    The answer will be the HCF of the three numbers. (41 in this case)

16. Which of the following represents the largest 4 digit number which can be added to 7249 in order to make the derived number divisible by each of 12, 14, 21, 33, and 54.

    (a) 9123 (b) 9383
    (c) 8727 (d) None of these

    Answer: The LCM of the numbers 12, 14, 21, 33 and 54 is 8316. Hence, in order for the condition to be satisfied we need to get the number as:

    $$7249 + n = 8316 \times 2$$

    Hence, $n = 9383$.

17. Find the greatest number of 5 digits, that will give us a remainder of 5, when divided by 8 and 9 respectively.

    (a) 99931 (b) 99941
    (c) 99725 (d) None of these

    Answer: The LCM of 8 and 9 is 72. The largest 5 digit multiple of 72 is 99936. Hence, the required answer is 99941.

18. The least perfect square number which is divisible by 3, 4, 6, 8, 10 and 11 is:

    Solution: The number should have at least one 3, three 2's, one 5 and one 11 for it to be divisible by 3, 4, 6, 8, 10 and 11.

    Further, each of the prime factors should be having an even power. Thus, the correct answer will be: $3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 11$

19. Find the greatest number of four digits which when divided by 10, 11, 15 and 22 leaves 3, 4, 8 and 15 as remainders respectively.

    (a) 9907 (b) 9903
    (c) 9893 (d) None of these

    Answer: First find the greatest 4 digit multiple of the LCM of 10, 11, 15 and 22. (In this case it is 9900). Then, subtract 7 from it to give the answer.

20. Find the HCF of $(3^{125} - 1)$ and $(3^{35} - 1)$.

    Answer: The solution of this question is based on the rule that:

    The HCF of $(a^m - 1)$ and $(a^n - 1)$ is given by $(a^{\text{HCF of m, n}} - 1)$

    Thus, in this question the answer is: $(3^5 - 1)$. Since 5 is the HCF of 35 and 125.

21. What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

    (a) 7 (b) 8
    (c) 22 (d) None of these

22. Find the greatest number, which will divide 215, 167 and 135 so as to leave the same remainder in each case.

    (a) 64 (b) 32
    (c) 24 (d) 16

23. Find the L.C.M of 2.5, 0.5 and 0.175
    (a) 2.5 (b) 5
    (c) 7.5 (d) 17.5
24. The L.C.M of 4.5; 0.009; and 0.18 = ?
    (a) 4.5 (b) 45
    (c) 0.225 (d) 2.25
25. The L.C.M of two numbers is 1890 and their H.C.F is 30. If one of them is 270, the other will be
    (a) 210 (b) 220
    (c) 310 (d) 320
26. What is the smallest number which when increased by 6 is divisible by 36, 63 and 108?
    (a) 750 (b) 752
    (c) 754 (d) 756
27. The smallest square number, which is exactly divisible by 2, 3, 4, – 9, 6, 18, 36 and 60, is
    (a) 900 (b) 1600
    (c) 3600 (d) None of these
28. The H.C.F of two numbers is 11, and their L.C.M is 616. If one of the numbers is 88, find the other?
    (a) 77 (b) 87
    (c) 97 (d) None of these
29. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes?
    (a) 11m/m (b) 13 m/m
    (c) 17 m/m (d) None of these
30. The HCF and LCM of two numbers are 12 and 144 respectively. If one of the numbers is 36, the other number is
    (a) 4 (b) 48
    (c) 72 (d) 432

Answers

21. (c)	22. (d)	23. (d)	24. (a)	25. (a)
26. (a)	27. (a)	28. (a)	29. (c)	30. (b)

DIVISIBILITY

A number x is said to be divisible by another number 'y' if it is completely divisible by Y (i.e. it should leave no remainder).

In general it can be said that any integer I, when divided by a natural number N, there exist a unique pair of numbers Q and R which are called the quotient and Remainder respectively.

Thus, $I = QN + R$.

For any integer I and any natural number n there is a unique pair of numbers a and b such that:

$$I = QN + R$$

Where Q is an integer and N is a natural number or zero and $0 \geq R < N$ (i.e. remainder has to be a whole number less than N.)

If the remainder is zero we say that the number I is divisible by N.

When $R \neq 0$, we say that the number I is divisible by N with a remainder.

Thus, 25/8 can be written as: 25 = 3.8 + 1 (3 is the quotient and 1is the remainder)

While, –25/7 will be written as –25 = 7· (–4) + 3 (–4 is the quotient and 3 is the remainder)

Note: An integer $b \neq 0$ is said to divide an integer a if there exists another integer c such that:

$$a = bc$$

It is important to explain at this point a couple of concepts with respect to the situation, when we divide a negative number by a natural number N.

Suppose, we divide – 32 by 7. Contrary to what you might expect the remainder in this case is + 3 (and not – 4). This is because the remainder is always non negative.

Thus, –32/7 gives quotient as – 5 and remainder as + 3.

The relationship between the remainder and the decimal:

1. Suppose we divide 42 by 5. The result has a quotient of 8 and remainder of 2.

   But 42/5 = 8.4. As you can see, the answer has an integer part and a decimal part. The integer part being 8 (equals the quotient), the decimal part is 0.4 (and is given by 2/5).

   Since, we have also seen that for any divisor N, the set of remainders obeys the inequality $0 \leq R < N$, we should realise that any divisor N, will yield exactly N possible remainders. (For example If the divisor is 3, we have 3 possible remainders 0, 1 and 2. Further, when 3 is the divisor we can have only 3 possible decimal values .00, .333 & 0.666 corresponding to remainders of 0, 1 or 2. I would want you to remember this concept when you study the fraction to percentage conversion table in the chapter of percentages (Table 5.2)).
2. In the case of –42 being divided by 5, the value is – 8.4. In this case the interpretation should be thus: The integer part is – 9 (which is also the quotient of this division) and the decimal part is 0.6 (corresponding to 3/5) Notice that since the remainder cannot be negative, the decimal too cannot be negative.

Theorems of Divisibility

(a) If a is divisible by b then ac is also divisible by b.

(b) If a is divisible by b and b is divisible by c then a is divisible by c.

(c) If a and b are natural numbers such that a is divisible by b and b is divisible by a then $a = b$.

(d) If n is divisible by d and m is divisible by d then $(m + n)$ and $(m - n)$ are both divisible by d. This has an important implication. Suppose 28 and 742 are both divisible by 7. Then (742 + 28) as well as (742 – 28) are divisible by 7. (and in fact so is + 28 – 742).

(e) If a is divisible by b and c is divisible by d then ac is divisible by bd.

(f) The highest power of a prime number p, which divides $n!$ exactly is given by

$$[n/p] + [n/p^2] + [n/p^3] + \dots$$

where $[x]$ denotes the greatest integer less than or equal to x.

As we have already seen earlier –

Any composite number can be written down as a product of its prime factors. (Also called standard form)

Thus, for example the number 1240 can be written as $2^3 \times 31^1 \times 5^1$.

The standard form of any number has a huge amount of information stored in it. The best way to understand the information stored in the standard form of a number is to look at concrete examples. As a reader I want you to understand each of the processes defined below and use them to solve similar questions given in the exercise that follows and beyond:

1. Using the standard form of a number to find the sum and the number of factors of the number:

(a) Sum of factors of a number:

Suppose, we have to find the sum of factors and the number of factors of 240.

$$240 = 2^4 \times 3^1 \times 5^1$$

The sum of factors will be given by:

$$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1) = 31 \times 4 \times 6 = 744$$

Note: This is a standard process, wherein you create the same number of brackets as the number of distinct prime factors the number contains and then each bracket is filled with the sum of all the powers of the respective prime number starting from 0 to the highest power of that prime number contained in the standard form.

Thus, for 1240, we create 3 brackets—one each for 2, 3 and 5. Further in the bracket corresponding to 2 we write $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)$.

Hence, for example for the number $40 = 2^3 \times 5^1$, the sum of factors will be given by: $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ {2 brackets since 40 has 2 distinct prime factors 2 and 5}

(b) Number of factors of the number:

Let us explore the sum of factors of 40 in a different context.

$$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$$
$$= 2^0 \times 5^0 + 2^0 \times 5^1 + 2^1 \times 5^0 + 2^1 \times 5^1 + 2^2 \times 5^0 + 2^2 \times 5^1 + 2^3 \times 5^0 + 2^3 \times 5^1$$
$$= 1 + 5 + 2 + 10 + 4 + 20 + 8 + 40 = 90$$

A clear look at the numbers above will make you realize that it is nothing but the addition of the factors of 40

Hence, we realise that the number of terms in the expansion of $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ will give us the number of factors of 40. Hence, 40 has $4 \times 2 = 8$ factors.

Note: The moment you realise that $40 = 2^3 \times 5^1$ the answer for the number of factors can be got by $(3 + 1)(1 + 1) = 8$

2. Sum and Number of even and odd factors of a number.

Suppose, you are trying to find out the number of factors of a number represented in the standard form by: $2^3 \times 3^4 \times 5^2 \times 7^3$

As you are already aware the answer to the question is $(3 + 1)(4 + 1)(2 + 1) \times (3 + 1)$ and is based on the logic that the number of terms will be the same as the number of terms in the expansion: $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$.

Now, suppose you have to find out the sum of the even factors of this number. The only change you need to do in this respect will be evident below. The answer will be given by:

$(2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

Note: That we have eliminated 2^0 from the original answer. By eliminating 2^0 from the expression for the sum of all factors you are ensuring that you have only even numbers in the expansion of the expression.

Consequently, the number of even factors will be given by: $(3)(4 + 1)(2 + 1)(3 + 1)$

i.e. Since 2^0 is eliminated, we do not add 1 in the bracket corresponding to 2.

Let us now try to expand our thinking to try to think about the number of odd factors for a number.

In this case, we just have to do the opposite of what we did for even numbers. The following step will make it clear:

Odd factors of the number whose standard form is : $2^3 \times 3^4 \times 5^2 \times 7^3$

Sum of odd factors = $(2^0)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

i.e.: Ignore all powers of 2. The result of the expansion of the above expression will be the complete set of odd factors of the number. Consequently, the number of odd factors for the number will be given by the number of terms in the expansion of the above expression.

Thus, the number of odd factors for the number $2^3 \times 3^4 \times 5^2 \times 7^3 = 1 \times (4 + 1)(2 + 1)(3 + 1)$.

3. **Sum and number of factors satisfying other conditions for any composite number**

These are best explained through examples:

(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.

Solution : $1200 = 2^4 \times 5^2 \times 3^1$.

For a factor to be divisible by 15 it should compulsorily have 3^1 and 5^1 in it. Thus, sum of factors divisible by 15 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (5^1 + 5^2)(3^1)$ and consequently the number of factors will be given by $5 \times 2 \times 1 = 10$.

(What we have done is ensure that in every individual term of the expansion, there is a minimum of $3^1 \times 5^1$. This is done by removing powers of 3 and 5 which are below 1.)

Task for the student: Physically verify the answers to the question above and try to convert the logic into a mental algorithm.

NOTE FROM THE AUTHOR—The need for thought algorithms:

I have often observed that the key difference between understanding a concept and actually applying it under examination pressure, is the presence or absence of a mental thought algorithm which clarifies the concept to you in your mind. The thought algorithm is a personal representation of a concept—and any concept that you read/understand in this book (or elsewhere) will remain an external concept till it remains in someone else's words. The moment the thought becomes internalised the concept becomes yours to apply and use.

Practice Exercise on Factors

For the number 2450 find.

1. The sum and number of all factors.
2. The sum and number of even factors.
3. The sum and number of odd factors.
4. The sum and number of factors divisible by 5
5. The sum and number of factors divisible by 35.
6. The sum and number of factors divisible by 245.

For the number 7200 find.

7. The sum and number of all factors.
8. The sum and number of even factors.
9. The sum and number of odd factors.
10. The sum and number of factors divisible by 25.
11. The sum and number of factors divisible by 40.
12. The sum and number of factors divisible by 150.
13. The sum and number of factors not divisible by 75.
14. The sum and number of factors not divisible by 24.
15. Find the number of divisors of 1728.
    (a) 18 (b) 30
    (c) 28 (d) 20
16. Find the number of divisors of 1080 excluding the throughout divisors, which are perfect squares.
    (a) 28 (b) 29
    (c) 30 (d) 31
17. Find the number of divisors of 544 excluding 1 and 544.
    (a) 12 (b) 18
    (c) 11 (d) 10
18. Find the number of divisors 544 which are greater than 3.
    (a) 15 (b) 10
    (c) 12 (d) None of these.
19. Find the sum of divisors of 544 excluding 1 and 544.
    (a) 1089 (b) 545
    (c) 589 (d) 1134
20. Find the sum of divisors of 544 which are perfect squares.
    (a) 32 (b) 64
    (c) 42 (d) 21
21. Find the sum of odd divisors of 544.
    (a) 18 (b) 34
    (c) 68 (d) 36
22. Find the sum of even divisors of 4096.

(a) 8192 (b) 6144
(c) 8190 (d) 6142

23. Find the sum the sums of divisors of 144 and 160.
(a) 589 (b) 781
(c) 735 (d) None of these

24. Find the sum of the sum of even divisors of 96 and the sum of odd divisors of 3600.
(a) 639 (b) 735
(c) 651 (d) 589

Answers

15. (c)	16. (a)	17. (d)	18. (b)	19. (c)
20. (d)	21. (a)	22. (c)	23. (b)	24. (c)

NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a product: $24 \times 32 \times 17 \times 23 \times 19 = (2^3 \times 3^1) \times (2^5) \times 17^1 \times 23 \times 19$.

As you can notice, this product will have no zeroes because it has no 5 in it.

However, if you have an expression like: $8 \times 15 \times 23 \times 17 \times 25 \times 22$

The above expression can be rewritten in the standard form as:

$$2^3 \times 3^1 \times 5^1 \times 23 \times 17 \times 5^2 \times 2^1 \times 11^1$$

Zeroes are formed by a combination of 2×5. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's that can be formed.

In the above product, there are four twos and two fives. Hence, we shall be able to form only two pairs of (2×5). Hence, there will be 2 zeroes in the product.

(Refer to Solved Example No. 1.11 for another example of this)

Finding the Number of Zeroes in a Factorial Value

Suppose you had to find the number of zeroes in 6!.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (3 \times 2) \times (5) \times (2 \times 2) \times (3) \times (2) \times (1)$.

The above expression will have only one pair of 5×2, since there is only one 5 and an abundance of 2's.

It is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's. The process for this is explained in Solved Examples 1.1 to 1.3.

EXERCISES FOR SELF-PRACTICE

Find the number of zeroes in the following cases:

1. 47!
2. 58!
3. $13 \times 15 \times 22 \times 125 \times 44 \times 35 \times 11$
4. $12 \times 15 \times 5 \times 24 \times 13 \times 17$
5. 173!
6. $144! \times 5 \times 15 \times 22 \times 11 \times 44 \times 135$
7. 148!
8. 1093!
9. 1132!
10. $1142! \times 348! \times 17!$

A special implication: Suppose you were to find the number of zeroes in the value of the following factorial values:

$$45!, 46!, 47!, 48!, 49!$$

What do you notice? The number of zeroes in each of the cases will be equal to 10. Why does this happen? It is not difficult to understand that the number of fives in any of these factorials is equal to 10. The number of zeroes will only change at 50! (It will become 12).

In fact, this will be true for all factorial values between two consecutive products of 5.

Thus, 50!, 51!, 52!, 53! And 54! will have 12 zeroes (since they all have 12 fives).

Similarly, 55!, 56!, 57!, 58! And 59! will each have 13 zeroes.

Apart from this fact, did you notice another thing? That while there are 10 zeroes in 49! there are directly 12 zeroes in 50!. This means that there is no value of a factorial which will give 11 zeroes. This occurs because to get 50! we multiply the value of 49! by 50. When you do so, the result is that we introduce two 5's in the product. Hence, the number of zeroes jumps by two (since we never had any paucity of twos.)

Note: at 124! you will get 24 + 4 => 28 zeroes.

At 125! you will get 25 + 5 + 1 = 31 zeroes. (A jump of 3 zeroes.)

EXERCISES FOR SELF-PRACTICE

1. n! has 23 zeroes. What is the maximum possible value of n?
2. n! has 13 zeroes. The highest and least values of n are?
3. Find the number of zeroes in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times \ldots \times 49^{49}$

4. Find the number of zeroes in:
$100^1 \times 99^2 \times 98^3 \times 97^4 \times \ldots\ldots\ldots \times 1^{100}$
5. Find the number of zeroes in:
$1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \times 5^{5!} \times \ldots\ldots 10^{10!}$
6. Find the number of zeroes in the value of:
$2^2 \times 5^4 \times 4^6 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
7. What is the number of zeroes in the following:
   (a) $3200 + 1000 + 40000 + 32000 + 15000000$
   (b) $3200 \times 1000 \times 40000 \times 32000 \times 15000000$

Solution:

1. This can never happen.
2. 59 and 55 respectively.
3. The fives will be less than the twos. Hence, we need to count only the fives.
Thus : $5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$
gives us: $5 + 10 + 15 + 20 + 25 + 25 + 30 + 35 + 40 + 45$ fives. Thus, the product has 250 zeroes.
4. Again the key here is to count the number of fives. This can get done by:
$100^1 \times 95^6 \times 90^{11} \times 85^{16} \times 80^{21} \times 75^{26} \times \ldots\ldots 5^{96}$
$(1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + \ldots\ldots + 96) + (1 + 26 + 51 + 76)$
$= 20 \times 48.5 + 4 \times 38.5$ (Using sum of A.P. explained in the next chapter.)
$= 970 + 154 = 1124.$
5. The answer will be the number of 5's. Hence, it will be $5! + 10!$
6. The number of fives is again lesser than the number of twos.
The number of 5's will be given by the power of 5 in the product:
$$5^4 \times 10^8 \times 15^{12} \times 20^{16} \times 10^{18} \times 25^{20}$$
$$= 4 + 8 + 12 + 16 + 18 + 40 = 98.$$
7. A. The number of zeroes in the sum will be two, since:

3200
1000
40000
32000
15076200
15152400

Thus, in such cases the number of zeroes will be the least number of zeroes amongst the numbers.
Exception: 3200 + 1800 = 5000 (three zeroes, not two).

B. The number of zeroes will be:
$$2 + 3 + 4 + 3 + 6 = 18.$$

An extension of the process for finding the number of zeroes.
Consider the following questions:

1. Find the highest power of 5 which is contained in the value of 127!
2. When 127! is divided by 5^n the result is an integer. Find the highest possible value for n.
3. Find the number of zeroes in 127!

In each of the above cases, the value of the answer will be given by:
$$[127/5] + [127/25] + [127/125]$$
$$= 25 + 5 + 1 = 31$$

This process can be extended to questions related to other prime numbers. For example:
Find the highest power of:

1. 3 which completely divides 38!
Solution: $[38/3] + [38/3^2] + 38/3^3] = 12 + 4 + 1 = 17$
2. 7 which is contained in 57!
$[57/7] + [57/7^2] = 8 + 1 = 9.$

This process changes when the divisor is not a prime number. You are first advised to go through worked out problems 1.4, 1.5, 1.6 and 1.19 .

Now try to solve the following exercise:

1. Find the highest power of 7 which divides 81!
2. Find the highest power of 42 which divides 122!
3. Find the highest power of 84 which divides 342!
4. Find the highest power of 175 which divides 344!
5. Find the highest power of 360 which divides 520!

Solution Hints:

1. You will check only with 7.
2. You will need to check with 7 only. (Since $42 = 7 \times 3 \times 2$)
3. $84 = 7 \times 3 \times 2 \times 2$. You will need to check for the number of 7's only.
4. $175 = 5^2 \times 7^1$. In this case you need to be careful. You will need to check the number of 5^2's and the number of 7's.
5. $36 = 2^3 \times 3^2 \times 5^1$. In this case you will need to check for the number of 2^3's, the number of 3^2's and the number of 5^1's.

EXERCISES FOR SELF-PRACTICE

1. Find the maximum value of n such that 157! is perfectly divisible by 10^n.

(a) 37 (b) 38
(c) 16 (d) – 1.15

2. Find the maximum value of n such that 157! is perfectly divisible by 12^n.
(a) 77 (b) 76
(c) 75 (d) 78

3. Find the maximum value of n such that 157! is perfectly divisible by 18^n.
(a) 37 (b) 38
(c) 39 (d) 40

4. Find the maximum value of n such that 50! is perfectly divisible by 2520^n.
(a) 6 (b) 8
(c) 7 (d) None of these

5. Find the maximum value of n such that 50! Is perfectly divisible by 12600^n.
(a) 7 (b) 6
(c) 8 (d) None of these

6. Find the maximum value of n such that 77! is perfectly divisible by 720^n.
(a) 35 (b) 18
(c) 17 (d) 36

7. Find the maximum value of n such that $42 \times 57 \times 92 \times 91 \times 52 \times 62 \times 63 \times 64 \times 65 \times 66 \times 67$ is perfectly divisible by 42^n.
(a) 4 (b) 3
(c) 5 (d) 6

8. Find the maximum value of n such that $570 \times 60 \times 30 \times 90 \times 100 \times 500 \times 700 \times 343 \times 720 \times 81$ is perfectly divisible by 30^n.
(a) 12 (b) 11
(c) 14 (d) 13

9. Find the maximum value of *n* such that $77 \times 42 \times 37 \times 57 \times 30 \times 90 \times 70 \times 2400 \times 2402 \times 243 \times 343$ is perfectly divisible by 21^n.
(a) 9 (b) 11
(c) 10 (d) 7

Find the number of consecutive zeros at the end of the following numbers.

10. 72!
(a) 17 (b) 9
(c) 8 (d) 16

11. $77! \times 42!$.
(a) 24 (b) 9
(c) 27 (d) 18

12. $100! + 200!$
(a) 73 (b) 24
(c) 11 (d) 22

13. $57 \times 60 \times 30 \times 15625 \times 4096 \times 625 \times 875 \times 975$
(a) 6 (b) 16
(c) 17 (d) 15

14. $1! \times 2! \times 3! \times 4! \times 5! \times$ - - - - - - - $\times 50!$
(a) 235 (b) 12
(c) 262 (d) 105

15. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times 7^7 \times 8^8 \times 9^9 \times 10^{10}$.
(a) 25 (b) 15
(c) 10 (d) 20

16. $100! \times 200!$
(a) 49 (b) 73
(c) 132 (d) 33

Answers

1. (b)	2. (c)	3. (a)	4. (b)	5. (b)
6. (c)	7. (b)	8. (b)	9. (d)	10. (d)
11. (c)	12. (b)	13. (d)	14. (c)	15. (b)
16. (b)				

Co-Prime or Relatively Prime Numbers Two or more numbers that do not have a common factor are known as co-prime or relatively prime. In other words, these numbers have a highest common factor of unity.

If two numbers *m* and *n* are relatively prime and the natural number *x* is divisible by both *m* and *n* independently then the number *x* is also divisible by *mn*.

Key Concept 1: The spotting of two numbers as co-prime has a very important implication in the context of the two numbers being in the denominators of fractions.

The concept is again best understood through an example:

Suppose, you are doing an operation of the following format – $M/8 + N/9$ where M & N are integers.

What are the chances of the result being an integer, if M is not divisible by 8 and N is not divisible by 9? A little bit of thought will make you realise that the chances are zero. The reason for this is that 8 and 9 are co-prime and the decimals of co-prime numbers never match each other.

Note: this will not be the case in the case of:

$$M/3 + N/27.$$

In this case even if 3 and 27 are not dividing M and N respectively, there is a possibility of the values of M and N being such that you have an integral answer.

For instance: $5/3 + 36/27 = 81/27 = 3$

The result will never be integral if the two denominators are co-prime.

Note: This holds true even for expressions of the nature $A/7 - B/6$ etc.

This has huge implications for problem solving especially in the case of solving linear equations related to word based problems. Students are advised to try to use these throughout Block 1, 2 and 3 of this book.

Key Concept 2: Two consecutive integers are always co-prime.

Example: Find all five-digit numbers of the form $34X5Y$ that are divisible by 36.

Solution: 36 is a product of two co-primes 4 and 9. Hence, if $34X5Y$ is divisible by 4 and 9, it will also be divisible by 36. Hence, for divisibility by 4, we have that the value of Y can be 2 or 6. Also, if Y is 2 the number becomes $34X52$. For this to be divisible by 9, the addition of $3 + 4 + X + 5 + 2$ should be divisible by 9. For this X can be 4.

Hence the number 34452 is divisible by 36.

Also for $Y = 6$, the number 34×56 will be divisible by 36 when the addition of the digits is divisible by 9. This will happen when X is 0 or 9. Hence, the numbers 34056 and 34956 will be divisible by 36.

EXERCISES FOR SELF-PRACTICE

Find all numbers of the form $56x3y$ that are divisible by 36.
Find all numbers of the form $72xy$ that are divisible by 45.
Find all numbers of the form $135xy$ that are divisible by 45.
Find all numbers of the form $517xy$ that are divisible by 89.

Divisibility Rules

Divisibility by 2 or 5: A number is divisible by 2 or 5 if the last digit is divisible by 2 or 5.

Divisibility by 3 (or 9): All such numbers the sum of whose digits are divisible by 3 (or 9) are divisible by 3 (or 9).

Divisibility by 4: A number is divisible by 4 if the last 2 digits are divisible by 4.

Divisibility by 6: A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 8: A number is divisible by 8 if the last 3 digits of the number are divisible by 8.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits in the odd places and the sum of the digits in the even place is zero or is divisible by 11.

Divisibility by 12: All numbers divisible by 3 and 4 are divisible by 12.

Divisibility by 7, 11 or 13: The integer n is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1000 is divisible by 7, 11 or 13.

Example: 473312 is divisible by 7 since the difference between $473 - 312 = 161$ is divisible by 7.

Even Numbers: All integers that are divisible by 2 are even numbers. They are also denoted by $2n$.

Example: 2, 4, 6, 12, 122, –2, –4, –12. Also note that zero is an even number.
2 is the lowest positive even number.

Odd Numbers: All integers that are not divisible by 2 are odd numbers. Odd number leave a remainder of 1 on being divided by 2. They are denoted by $2n + 1$ or $2n - 1$.

Lowest positive odd number is 1.

Example: –1, –3, –7, –35, 3, 11 etc.

Complex Numbers: The arithmetic combination of real numbers and imaginary numbers are called complex numbers.

Alternately: All numbers of the form $a + ib$, where $i = \sqrt{-1}$ are called complex number.

Twin Primes: A pair of prime numbers are said to be twin prime when they differ by 2.

Example: 3 and 5 are Twin Primes, so also are 11 and 13.

Perfect Numbers: A number n is said to be a perfect number if the sum of all the divisors of n (including n) is equal to $2n$.

Example: $6 = 1 \times 2 \times 3$ sum of the divisors $= 1 + 2 + 3 + 6 = 12 = 2 \times 6$

$$28 = 1, 2, 4, 7, 14, 28, = 56 = 2 \times 28$$

Task for student: Find all perfect numbers below 1000.
Mixed Numbers: A number that has both an integral and a fractional part is known as a mixed number.
Triangular Numbers: A number which can be represented as the sum of consecutive natural numbers starting with 1 are called as triangular numbers.

e.g.: $1 + 2 + 3 + 4 = 10$

Certain Rules

1. Of n consecutive whole numbers $a, a + 1...a + n - 1$, one and only one is divisible by n.
2. *Mixed numbers:* A number that has both the integral and fractional part is known as mixed number.
3. If a number n can be represented as the product of two numbers p and q, that is, $n = p \cdot q$, then we say that the number n is divisible by p and by q and each of the numbers p and q is a divisor of the number n.
4. Any number n can be represented in the decimal system of numbers as

   $$N = a_k 10^k + a_{k-1} 10^{k-1} + a ... + a_i 10 + a_0$$

   Example: 2738 can be written as: $2 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$.
5. 3^n will always have an even number of tens. (Example: 2 in 27,8 in 81,24 in 243,72 in 729 and so on.)
6. A sum of 5 consecutive whole numbers will always be divisible by 5.
7. The difference between 2 numbers—

   $(xy) - (yx)$—will be divisible by 9
8. The square of an odd number when divided by 8 will always leave a remainder of 1.
9. The product of 3 consecutive natural numbers is divisible by 6.
10. The product of 3 consecutive natural numbers the first of which is even is divisible by 24.
11. Products:

    Odd × odd = odd

    Odd × even = even

    Even × even = even
12. All numbers not divisible by 3 have the property that their square will have a remainder of 1 when divided by 3.
13. $(a^2 + b^2)/(b^2 + c^2) = (a^2/b^2)$ if $a/b = b/c$.
14. The product of any r consecutive integers (numbers) is divisible by $r!$
15. If m and n are two integers then $(m + n)!$ is divisible by $m!n!$
16. Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.
17. Any number written in the form $10^n - 1$ is divisible by 3 and 9.
18. If a numerical expression contains no parentheses, first the operations of the third stage (involution or raising a number to a power) are performed, then the operations of the second stage (multiplication and division) and, finally, the operations of the first stage (addition and subtraction) are performed. In this case the operations of one and the same stage are performed in the sequence indicated by the notation. If an expression contains parentheses, then the operation indicated in the parentheses are to be performed first and then all the remaining operations. In this case operations of the numbers in parentheses as well as standing without parentheses are performed in the order indicated above.

    If a fractional expression is evaluated, then the operations indicated in the numerator and denominator of the function are performed and the first result is divided by the second.
19. $(a)^n/(a + 1)$ leaves a remainder of

    a if n is odd

    1 if n is even
20. $(a + 1)^n/a$ will always give a remainder of 1.
21. For any natural number n, n^5 has the same units digit as n has.
22. For any natural number: $n^3 - n$ is divisible by 6.

THE REMAINDER THEOREM

Consider the following question:

$$17 \times 23.$$

Suppose you have to find the remainder of this expression when divided by 12.

We can write this as:

$$17 \times 23 = (12 + 5) \times (12 + 11)$$

Which when expanded gives us:

$$12 \times 12 + 12 \times 11 + 5 \times 12 + 5 \times 11$$

You will realise that, when this expression is divided by 12, the remainder will only depend on the last term above:

Thus, $\frac{12 \times 12 + 12 \times 11 + 5 \times 12 + 5 \times 11}{12}$ gives the same remainder as $\frac{5 \times 11}{12}$

Hence, 7.

This is the remainder when 17×23 is divided by 12.

Learning Point: In order to find the remainder of 17×23 when divided by 12, you need to look at the individual remainders of 17 and 23 when divided by 12. The respective remainders (5 and 11) will give you the remainder of the original expression when divided by 12.

Mathematically, this can be written as:

The remainder of the expression $[A \times B \times C + D \times E]/M$, will be the same as the remainder of the expression $[A_R \times B_R \times C_R + D_R \times E_R]/M$.

Where A_R is the remainder when A is divided by M,

B_R is the remainder when B is divided by M,

C_R is the remainder when C is divided by M

D_R is the remainder when D is divided by M and

E_R is the remainder when E is divided by M,

We call this transformation as the remainder theorem transformation and denote it by the sign $\xrightarrow{R}$

Thus, the remainder of

$1421 \times 1423 \times 1425$ when divided by 12 can be given as:

$$\frac{1421 \times 1423 \times 1425}{12} \xrightarrow{R} \frac{5 \times 7 \times 9}{12} = \frac{35 \times 9}{12}$$

$$\xrightarrow{R} \frac{11 \times 9}{12}.$$

$\xrightarrow{R}$ gives us a remainder of 3.

In the above question, we have used a series of remainder theorem transformations (denoted by $\xrightarrow{R}$) and equality transformations to transform a difficult looking expression into a simple expression.

Try to solve the following questions on Remainder theorem:

Find the remainder in each of the following cases:

1. $17 \times 23 \times 126 \times 38$ divided by 8.
2. $243 \times 245 \times 247 \times 249 \times 251$ divided by 12.
3. $\frac{173 \times 261}{13} + \frac{248 \times 249 \times 250}{15}$.
4. $\frac{1021 \times 2021 \times 3021}{14}$.
5. $\frac{37 \times 43 \times 51}{7} + \frac{137 \times 143 \times 151}{9}$.

USING NEGATIVE REMAINDERS

Consider the following question:

Find the remainder when: 14×15 is divided by 8.

The obvious approach in this case would be

$$\frac{14 \times 15}{8} \xrightarrow{R} \frac{6 \times 7}{8} = \frac{42}{8} \xrightarrow{R} 2 \text{ (Answer)}.$$

However there is another option by which you can solve the same question:

When 14 is divided by 8, the remainder is normally seen as + 6. However, there might be times when using the negative value of the remainder might give us more convenience. Which is why you should know the following process:

Concept Note: Remainders by definition are always non-negative. Hence, even when we divide a number like – 27 by 5 we say that the remainder is 3 (and not – 2). However, looking at the negative value of the remainder has its own advantages in Mathematics as it results in reducing calculations.

Thus, when a number like 13 is divided by 8, the remainder being 5, the negative remainder is – 3.

(**Note:** It is in this context that we mention numbers like 13, 21, 29 etc as $8n + 5$ or $8n - 3$ numbers.)

Thus $\frac{14 \times 15}{8}$ will give us $\frac{-2 \times -1}{8}$ $R \rightarrow 2$.

Consider the advantage this process will give you in the following question:

$$\frac{51 \times 52}{53} \xrightarrow{R} \frac{-2 \times -1}{53} \xrightarrow{R} 2.$$

(The alternative will involve long calculations. Hence, the principle is that you should use negative remainders wherever you can. They can make life much simpler!!!)

What If the Answer Comes Out Negative

For instance, $\frac{62 \times 63 \times 64}{66}$ $R \rightarrow \frac{-4 \times -3 \times -2}{66}$ $R \rightarrow \frac{-24}{66}$.

But, we know that a remainder of –24, equals a remainder of 42 when divided by 66. Hence, the answer is 42.

Of course nothing stops you from using positive and negative remainders at the same time in order to solve the same question –

Thus $\frac{17 \times 19}{9}$ $R \rightarrow \frac{(-1) \times (1)}{9}$ $R \rightarrow -1$ $R \rightarrow 8$.

Dealing with Large Powers There are two tools which are effective in order to deal with large powers –

(A) If you can express the expression in the form $\frac{(ax+1)^n}{a}$, the remainder will become 1 directly. In such a case, no matter how large the value of the power n is, the remainder is 1.

For instance, $\frac{(37^{12635})}{9} \xrightarrow{R} \frac{(1^{12635})}{9} \xrightarrow{R} 1$.

In such a case the value of the power does not matter.

(B) $\frac{(mx-1)^n}{a}$. In such a case using -1 as the remainder it will be evident that the remainder will be +1 if n is even and it will be -1 (Hence $a-1$) when n is odd.

e.g.: $\frac{31^{127}}{8} \xrightarrow{R} \frac{(-1)^{127}}{8} \xrightarrow{R} \frac{(-1)}{8} \xrightarrow{R} 7$

ANOTHER IMPORTANT POINT

Suppose you were asked to find the remainder of 14 divided by 4. It is clearly visible that the answer should be 2.

But consider the following process:

$14/4 = 7/2 \xrightarrow{R} 1$ (The answer has changed!!)

What has happened?

We have transformed 14/4 into 7/2 by dividing the numerator and the denominator by 2. The result is that the original remainder 2 is also divided by 2 giving us 1 as the remainder. In order to take care of this problem, we need to reverse the effect of the division of the remainder by 2. This is done by multiplying the final remainder by 2 to get the correct answer.

Note: *In any question on remainder theorem, you should try to cancel out parts of the numerator and denominator as much as you can, since it directly reduces the calculations required.*

AN APPLICATION OF REMAINDER THEOREM

Finding the last two digits of an expression:

Suppose you had to find the last 2 digits of the expression:

$$22 \times 31 \times 44 \times 27 \times 37 \times 43$$

The remainder the above expression will give when it is divided by 100 is the answer to the above question.

Hence, to answer the question above find the remainder of the expression when it is divided by 100.

Solution: $\frac{22 \times 31 \times 44 \times 27 \times 37 \times 43}{100}$

$= \frac{22 \times 31 \times 11 \times 27 \times 37 \times 43}{25}$ (on dividing by 4)

$\xrightarrow{R} \frac{22 \times 6 \times 11 \times 2 \times 12 \times 18}{25} = \frac{132 \times 22 \times 216}{25}$

$\xrightarrow{R} \frac{7 \times 22 \times 16}{25}$

$= \frac{154 \times 16}{25} \xrightarrow{R} \frac{4 \times 16}{25} \xrightarrow{R} 14$

Thus the remainder being 14, (after division by 4). The actual remainder should be 56.

[Don't forget to multiply by 4 !!]

Hence, the last 2 digits of the answer will be 56.

Using negative remainders here would have helped further.

Note: Similarly finding the last three digits of an expression means finding the remainder when the expression is divided by 1000.

EXERCISES

1. Find the remainder when 73 + 75 + 78 + 57 + 197 is divided by 34.
   (a) 32 (b) 4
   (c) 15 (d) 28
2. Find the remainder when $73 \times 75 \times 78 \times 57 \times 197$ is divided by 34.
   (a) 22 (b) 30
   (c) 15 (d) 28
3. Find the remainder when $73 \times 75 \times 78 \times 57 \times 197 \times 37$ is divided by 34.
   (a) 32 (b) 30
   (c) 15 (d) 28
4. Find the remainder when 43^{197} is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1
5. Find the remainder when 51^{203} is divided by 7.
   (a) 4 (b) 2
   (c) 1 (d) 6
6. Find the remainder when 59^{28} is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1
7. Find the remainder when 67^{99} is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1

8. Find the remainder when 75^{80} is divided by 7.
(a) 4 (b) 3
(c) 2 (d) 6
9. Find the remainder when 41^{77} is divided by 7.
(a) 2 (b) 1
(c) 6 (d) 4
10. Find the remainder when 21^{875} is divided by 17.
(a) 8 (b) 13
(c) 16 (d) 9
11. Find the remainder when 54^{124} is divided by 17.
(a) 4 (b) 5
(c) 13 (d) 15
12. Find the remainder when 83^{261} is divided by 17.
(a) 13 (b) 9
(c) 8 (d) 2
13. Find the remainder when 25^{102} is divided by 17.
(a) 13 (b) 15
(c) 4 (d) 2

Answers

1. (b)	2. (a)	3. (a)	4. (d)	5. (a)
6. (b)	7. (d)	8. (a)	9. (c)	10. (b)
11. (a)	12. (d)	13. (c)		

Units Digit

(A) By the logic of what we have just seen above, the unit's digit of an expression will be got by getting the remainder when the expression is divided by 10.

Thus for example if we have to find the units digit of the expression:

$$17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63$$

We try to find the remainder –

$$\frac{17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63}{10}$$

$$\xrightarrow{R} \frac{7 \times 2 \times 6 \times 4 \times 7 \times 3}{10}$$

$$= \frac{14 \times 24 \times 21}{10} \xrightarrow{R} \frac{4 \times 4 \times 1}{10} = \frac{16}{10} \xrightarrow{R} 6.$$

Hence, the required answer is 6.

This could have been directly got by multiplying: $7 \times 2 \times 6 \times 4 \times 7 \times 1 \times 3$ and only accounting for the units' digit.

(B) Unit's digits in the contexts of powers –

Study the following table carefully.

Unit's digit when 'N' is raised to a power

Number Ending With	*Value of power* 1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2
3	3	9	7	1	3	9	7	1	3
4	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7
8	8	4	2	6	8	4	2	6	8
9	9	1	9	1	9	1	9	1	9
0	0	0	0	0	0	0	0	0	0

In the table above, if you look at the columns corresponding to the power 5 or 9 you will realize that the unit's digit for all numbers is repeated (i.e. it is 1 for 1, 2 for, 3 for 3….9 for 9.)

This means that whenever we have any number whose unit's digit is 'x' and it is raised to a power of the form $4n + 1$, the value of the unit's digit of the answer will be the same as the original units digit.

Illustrations:

$(1273)^{101}$ will give a unit's digit of 3. $(1547)^{25}$ will give a units digit of 7 and so forth.

Thus, the above table can be modified into the form –

Value of Power

Number ending in	*'N'* *4n + 1*	*4n + 2*	*4n + 3*	$U_{n'}$
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

[Remember, at this point that we had said (in the back to school section of Block 1) that all natural numbers can be expressed in the form $4n + x$. Hence, with the help of the logic that helps us build this table, we can easily derive the units digit of any number when it is raised to a power.)

A special Case

Question:

What will be the Unit's digit of $(1273)^{122!}$?

Solution:

122! is a number of the form $4n$. Hence, the answer should be 1. [**Note:** 1 here is derived by thinking of it as 3 (for $4n + 1$), 9 (for $4n + 2$), 7 (for $4n + 3$), 1(for $4n$)]

■ EXERCISE FOR PRACTICE ■

Find the Units digit in each of the following cases:

1. $2^2 \times 4^4 \times 6^6 \times 8^8$
2. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \ldots. \times 100^{100}$
3. $17 \times 23 \times 51 \times 32 + 15 \times 17 \times 16 \times 22$
4. $13 \times 17 \times 22 \times 34 + 12 \times 6 \times 4 \times 3 - 13 \times 33$
5. $37^{123} \times 43^{144} \times 57^{226} \times 32^{127} \times 52^{5!}$
6. $67 \times 37 \times 43 \times 91 \times 42 \times 33 \times 42$
   (a) 2 (b) 6
   (c) 8 (d) 4
7. $67 \times 35 \times 43 \times 91 \times 47 \times 33 \times 49$
   (a) 1 (b) 9
   (c) 5 (d) 6
8. $67 \times 35 \times 45 \times 91 \times 42 \times 33 \times 81$
   (a) 2 (b) 4
   (c) 0 (d) 8
9. $67 \times 35 \times 45 + 91 \times 42 \times 33 \times 82$
   (a) 8 (b) 7
   (c) 0 (d) 5
10. $(52)^{97} \times (43)^{72}$
    (a) 2 (b) 6
    (c) 8 (d) 4
11. $(55)^{75} \times (93)^{175} \times (107)^{275}$
    (a) 7 (b) 3
    (c) 5 (d) 0
12. $(173)^{45} \times (152)^{77} \times (777)^{999}$
    (a) 2 (b) 4
    (c) 8 (d) 6
13. $81 \times 82 \times 83 \times 84 \times 86 \times 87 \times 88 \times 89$
    (a) 0 (b) 6
    (c) 2 (d) 4
14. $82^{43} \times 83^{44} \times 84^{97} \times 86^{98} \times 87^{105} \times 88^{94}$
    (a) 2 (b) 6
    (c) 4 (d) 8
15. $432 \times 532 + 532 \times 974 + 537 \times 531 + 947 \times 997$
    (a) 5 (b) 6
    (c) 9 (d) 8

Answers

6. (d)	7. (c)	8. (c)	9. (b)	10. (a)
11. (c)	12. (c)	13. (b)	14. (b)	15. (d)

WORKED-OUT PROBLEMS

Problem 1.1 Find the number of zeroes in the factorial of the number 18.

Solution 18! Contains 15 and 5, which combined with one even number give zeroes. Also, 10 is also contained in 18!, which will give an additional zero. Hence, 18! Contains 3 zeroes and the last digit will always be zero.

Problem 1.2 Find the numbers of zeroes in 27!

Solution $27! = 27 \times 26 \times 25 \times \ldots \times 20 \times \ldots \times 15 \times \ldots \times 10 \times \ldots \times 5 \times \ldots \times 1$.

A zero can be formed by combining any number containing 5 multiplied by any even number. Similarly, everytime a number ending in zero is found in the product, it will add an additional zero. For this problem, note that $25 = 5 \times 5$ will give 2 zeroes and zeroes will also be got by 20, 15, 10 and 5. Hence 27! Will have 6 zeroes.

Short-cut method: Number of zeroes is $27! \rightarrow [27/5] + [27/25]$ where $[x]$ indicates the integer just lower than the fraction Hence, $[27/5] = 5$ and $[27/5^2] = 1$, 6 zeroes

Problem 1.3 Find the number of zeroes in 137!

Solution $[137/5] + [137/5^2] + [137/5^3]$

$$= 27 + 5 + 1 = 33 \text{ zeroes}$$

(since the restriction on the number of zeroes is due to the number of fives.)

■ EXERCISE FOR SELF-PRACTICE ■

Find the number of zeroes in

(a) 81! (b) 100! (c) 51!

Answers

(a) 19 (b) 24 (c) 12

Problem 1.4 What exact power of 5 divides 87!

Solution [87/5] + [87/25] = 17 + 3 = 20

Problem 1.5 What power of 8 exactly divides 25!?

Solution If 8 were a prime number, the answer should be [25/8] = 3. But since 8 is not prime, use the following process.

The prime factors of 8 is $2 \times 2 \times 2$. For divisibility by 8, we need three twos. So, everytime we can find 3 twos, we add one to the power of 8 that divides 25! To count how we get 3 twos, we do the following. All even numbers will give one 'two' at least [25/2] = 12

Also, all numbers in 25! divisible by 2^2 will give an additional two $[25/2^2] = 6$

Further, all numbers in 25! divisible by 2^3 will give a third two. Hence $[25!/2^3] = 3$

And all numbers in 25! divisible by 2^4 will give a fourth two. Hence $[25!/2^4] = 1$

Hence, total number of twos in 25! is 22. For a number to be divided by 8, we need three twos. Hence, since 25! has 22 twos, it will be divided by 8 seven times.

Problem 1.6 What power of 15 divides 87! exactly?

Solution $15 = 5 \times 3$. Hence, everytime we can form a pair of one 5 and one 3, we will count one.

87! contains – $[87/5] + [87/5^2] = 17 + 3 = 20$ fives

Also 87! contains – $[87/3] + [87/3^2] + [87/3^3] + [87/3^4] = 29 + \ldots$(more than 20 threes).

Hence, 15 will divide 87! twenty times since the restriction on the power is because of the number of 5s and not the number of 3s.

In fact, it is not very difficult to see that in the case of all the factors being prime, we just have to look for the highest prime number to provide the restriction for the power of the denominator.

Hence, in this case we did not need to check for anything but the number of 5s.

EXERCISES FOR SELF-PRACTICE

(a) What power of 30 will exactly divide 128!

Hints: $[128/5] + [128/5^2] + [128/5^3]$

(b) What power of 210 will exactly divide 142!

Problem 1.7 Find the last digit in the expression $(36472)^{123!} \times (34767)^{76!}$.

Solution If we try to formulate a pattern for 2 and its powers and their units digit, we see that the units digit for the powers of 2 goes as: 2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6 and so on. The number 2 when raised to a power of $4n + 1$ will always give a units digit of 2. This also means that the units digit for 2^{4n} will always end in 6. The power of 36472 is 123! . 123! can be written in the form $4n$. Hence, $(36472)^{123!}$ will end in 6.

The second part of the expression is $(34767)^{76!}$. The units digit depends on the power of 7. If we try to formulate a pattern for 7 and its powers and their units digit, we see that the units digit for the powers of 7 go as: 7 9 3 1 7 9 3 1 and so on. This means that the units digit of the expression 7^{4th} will always be 1.

Since 76! Can be written as a multiple of 4 as $4n$, we can conclude that the unit's digit in $(34767)^{76!}$ is 1.

Hence the units digit of $(36472)^{123!} \times (34767)^{76!}$ will be 6.

Counting

Problem 1.8 Find the number of numbers between 100 to 200 if

(i) Both 100 and 200 are counted.

(ii) Only one of 100 and 200 is counted.

(iii) Neither 100 nor 200 is counted.

Solution

(i) Both ends included-Solution: 200 – 100 + 1 = 101

(ii) One end included-Solution: 200 – 100 = 100

(iii) Both ends excluded-Solution: 200 – 100 – 1 = 99.

Problem 1.9 Find the number of even numbers between 122 and 242 if:

(i) Both ends are included.

(ii) Only one end is included.

(iii) Neither end is included.

Solution

(i) Both ends included-Solution: (242 – 122)/2 = 60 + 1 = 61

(ii) One end included-Solution: (242 – 122)/2 = 60

(iii) Both ends excluded-Solution: (242 – 122)/2 – 1 = 59

EXERCISES FOR SELF-PRACTICE

(a) Find the number of numbers between 140 to 259, both included, which are divisible by 7.

(b) Find the number of numbers between 100 to 200, that are divisible by 3.

Problem 1.10 Find the number of numbers between 300 to 400 (both included), that are not divisible by 2, 3, 4, and 5.

Solution Total numbers: 101

Step 1: Not divisible by 2 = All even numbers rejected: 51
Numbers left 50.

Step 2: Of which: divisible by 3 = first number 300, last number 399. But even numbers have already been removed, hence count out only odd numbers between 300 and 400 divisible by 3. This gives us that:
First number 303, last number 399, common difference 6

So, remove: [(399 – 303)/6] + 1 = 17.

∴ 50 – 17 = 33 numbers left.

We do not need to remove additional terms for divisibility by 4 since this would eliminate only even numbers (which have already been eliminated)

Step 3: Remove from 33 numbers left all odd numbers that are divisible by 5 and not divisible by 3.

Between 300 to 400, the first odd number divisible by 5 is 305 and the last is 395 (since both ends are counted, we have 10 such numbers as: [(395 – 305)/10 + 1 = 10].

However, some of these 10 numbers have already been removed to get to 33 numbers.

Operation left: Of these 10 numbers, 305, 315…395, reduce all numbers that are also divisible by 3. Quick perusal shows that the numbers start with 315 and have common difference 30.

Hence [(Last number – First number)/Difference + 1] = [(375 – 315)/30 + 1] = 3

These 3 numbers were already removed from the original 100. Hence, for numbers divisible by 5, we need to remove only those numbers that are odd, divisible by 5 but not by 3. There are 7 such numbers between 300 and 400.

So numbers left are: 33 – 7 = 26.

EXERCISES FOR SELF-PRACTICE

Find the number of numbers between 100 to 400 which are divisible by either 2, 3, 5 and 7.

Problem 1.11 Find the number of zeroes in the following multiplication: $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$.

Solution The number of zeroes depends on the number of fives and the number of twos. Here, close scrutiny shows that the number of twos is the constraint. The expression can be written as

$5 \times (5 \times 2) \times (5 \times 3) \times (5 \times 2 \times 2) \times (5 \times 5) \times (5 \times 2 \times 3) \times (5 \times 7) \times (5 \times 2 \times 2 \times 2) \times (5 \times 3 \times 3) \times (5 \times 5 \times 2)$

Number of 5s – 12, Number of 2s – 8.
Hence: 8 zeroes.

Problem 1.12 Find the remainder for $[(73 \times 79 \times 81)/11]$.

Solution The remainder for the expression: $[(73 \times 79 \times 81)/11]$ will be the same as the remainder for $[(7 \times 2 \times 4)/11]$

That is, $56/11 \Rightarrow$ remainder = 1

Problem 1.13 Find the remainder for $(3^{560}/8)$.

Solution $(3^{560}/8) = [(3^2)^{280}/8] = (9^{280}/8)$
$= [9.9.9\ldots(280 \text{ times})]/8$

remainder for above expression = remainder for $[1.1.1\ldots(280 \text{ times})]/8 \Rightarrow$ remainder = 1.

Problem 1.14 Find the remainder when $(2222^{5555} + 5555^{2222})/7$.

Solution This is of the form: $[(2222^{5555})/7 + (5555^{2222})/7]$

We now proceed to find the individual remainder of : $(2222^{5555})/7$. Let the remainder be R_1.

When 2222 is divided by 7, it leaves a remainder of 3.

Hence, for remainder purpose $(2222^{5555})/7 \approx (3^{5555}/7)$
$= (3.3^{5554})/7 = [3(3^2)^{2777}]/7 = [3.(7+2)^{2777}]/7 \approx (3.2^{2777})/7$
$= (3.2^2 \cdot 2^{2775})/7 = [3.2^2 \cdot (2^3)^{925}]/7$
$= [3.2^2 \cdot (8)^{925}]/7 \approx (12/7)$ Remainder = 5.

Similarly, $(5555^{2222})/7 \approx (4^{2222})/7 = [(2^2)^{2222}]/7 = (2)^{4444}/7 = (2.2^{4443})/7 = [2.(2^3)^{1481}]/7 = [2.(8)^{1481}]/7 \approx [2.(1)^{1481}]/7$ 2 (remainder).

Hence, $(2222^{5555})/7 + (5555^{2222})/7 = (5+2)/7 \Rightarrow$ Remainder $= 0$

Problem 1.15 Find the GCD and the LCM of the numbers 126, 540 and 630.

Solution The standard forms of the numbers are:

$126 \rightarrow 3 \times 3 \times 7 \times 2 \rightarrow 3^2 \times 7 \times 2$
$540 \rightarrow 3 \times 3 \times 3 \times 2 \times 2 \times 5 \rightarrow 2^2 \times 3^3 \times 5$
$630 \rightarrow 3 \times 3 \times 5 \times 2 \times 7 \rightarrow 2 \times 3^2 \times 5 \times 7$

For GCD we use Intersection of prime factors and the lowest power of all factors that appear in all three numbers. $2 \times 3^2 = 18$.

For LCM $\rightarrow$ Union of prime factors and highest power of all factors that appear in any one of the three numbers $\Rightarrow 2^2 \times 3^3 \times 5 \times 7 = 3780$.

EXERCISES FOR SELF-PRACTICE

Find the GCD and the LCM of the following numbers:

(i) 360, 8400 (ii) 120, 144
(iii) 275, 180, 372, 156 (iv) 70, 112
(v) 75, 114 (vi) 544, 720

Problem 1.16 The ratio of the factorial of a number x to the square of the factorial of another number, which when increased by 50% gives the required number, is 1.25. Find the number x.

(a) 6 (b) 5
(c) 9 (d) None of these

Solution Solve through options: Check for the conditions mentioned. When we check for option (a) we get $6! = 720$ and $(4!)^2 = 576$ and we have $6!/(4!)^2 = 1.25$, which is the required ratio.

Hence the answer is **(a)**

Problem 1.17 Three numbers A, B and C are such that the difference between the highest and the second highest two-digit numbers formed by using two of A, B and C is 5. Also, the smallest two two-digit numbers differ by 2. If $B > A > C$ then what is the value of B?

(a) 1 (b) 6 (c) 7 (d) 8

Solution Since B is the largest digit, option (a) is rejected. Check for option (b).

If B is 6, then the two largest two-digit numbers are 65 and 60 (Since, their difference is 5) and we have $B = 6$, $A = 5$ and $C = 0$.

But with this solution we are unable to meet the second condition. Hence (b) is not the answer. We also realise here that C cannot be 0.

Check for option (c).

B is 7, then the nos. are 76 and 71 or 75 and 70. In both these cases, the smallest two two-digit numbers do not differ by 2.

Hence, the answer is not (c).

Hence, option **(d)** is the answer

[To confirm, put $B = 8$, then the solution $A = 6$ and $C = 1$ satisfies the condition.]

Problem 1.18 Find the remainder when $2851 \times (2862)^2 \times (2873)^3$ is divided by 23.

Solution We use the remainder theorem to solve the problem. Using the theorem, we see that the following expressions have the same remainder.

$$\Rightarrow \frac{2851 \times (2862)^2 \times (2873)^3}{23}$$

$$\Rightarrow \frac{22 \times 10 \times 10 \times 21 \times 21 \times 21}{23}$$

$$\Rightarrow \frac{22 \times 8 \times 441 \times 21}{23} \Rightarrow \frac{22 \times 21 \times 8 \times 4}{23}$$

$$\Rightarrow \frac{462 \times 32}{23} \Rightarrow \frac{2 \times 9}{23} \Rightarrow \text{Remainder is } \mathbf{18}.$$

Problem 1.19 For what maximum value of n will the expression $\frac{10200!}{504^n}$ be an integer?

Solution For $\frac{10200!}{504^n}$ to be a integer, we need to look at the prime factors of 504 $\rightarrow$

$$504 = 3^2 \times 7 \times 8 = 2^3 \times 3^2 \times 7$$

We, thus have to look for the number of 7s, the number of 2^3s and the number of 3^2s that are contained in 10200!. The lowest of these will be the constraint value for n.

To find the number of 2^3s we need to find the number of 2s as

$$\left[\frac{10200}{2}\right] + \left[\frac{10200}{4}\right] + \left[\frac{10200}{8}\right] + \left[\frac{10200}{16}\right] + \left[\frac{10200}{32}\right]$$
$$+ \left[\frac{10200}{64}\right] + \left[\frac{10200}{128}\right] + \left[\frac{10200}{256}\right] + \left[\frac{10200}{512}\right] + \left[\frac{10200}{1024}\right]$$
$$+ \left[\frac{10200}{2048}\right] + \left[\frac{10200}{4096}\right] + \left[\frac{10200}{8192}\right]$$

where [ ] is the greatest integer function.

$= 5100 + 2550 + 1275 + 637 + 318 + 159 + 79 + 39 + 19 + 9 + 4 + 2 + 1$

Number of twos = 10192

Hence, number of $2^3 = 3397$

Similarly, we find the number of 3s as

$$\text{Number of threes} = \left[\frac{10200}{3}\right] + \left[\frac{10200}{9}\right] + \left[\frac{10200}{27}\right] + \left[\frac{10200}{81}\right] + \left[\frac{10200}{243}\right] + \left[\frac{10200}{729}\right] + \left[\frac{10200}{2187}\right] + \left[\frac{10200}{6561}\right]$$

$$= 3400 + 1133 + 377 + 125 + 41 + 13 + 4 + 1$$

Number of threes = 5094

$\therefore$ Number of 3^2 = 2547

Similarly we find the number of 7s as

$$\left[\frac{10200}{7}\right] + \left[\frac{10200}{49}\right] + \left[\frac{10200}{343}\right] + \left[\frac{10200}{2401}\right]$$

$$= 1457 + 208 + 29 + 4 = 1698.$$

Thus, we have, 1698 sevens, 2547 nines and 3397 eights contained in 10200!.

The required value of n will be given by the lowest of these three [The student is expected to explore why this happens]

Hence, answer = **1698.**

Short Cut We will look only for the number of 7s in this case. *Reason*: $7 > 3 \times 2$. So, the number of 7s must always be less than the number of 2^3.

And $7 > 2 \times 3$, so the number of 7s must be less than the number of 3^2.

Recollect that earlier we had talked about the finding of powers when the divisor only had prime factors. There we had seen that we needed to check only for the highest power as the restriction had to lie there.

In cases of the divisors having composite factors, we have to be slightly careful in estimating the factor that will reflect the restriction. In the above example, we saw a case where even though 7 was the lowest factor (in relation to 8 and 9), the restriction was still placed by 7 rather than by 9 (as would be expected based on the previous process of taking the highest number).

Problem 1.20 Find the units digit of the expression: $78^{5562} \times 56^{256} \times 97^{1250}$.

Solution We can get the units digits in the expression by looking at the patterns followed by 78, 56 and 97 when they are raised to high powers.

In fact, for the last digit we just need to consider the units digit of each part of the product.

A number (like 78) having 8 as the units digit will yield units digit as

$78^1 \to 8$	$78^5 \to 8$	$8^{4n+1} \to 8$
$78^2 \to 4$	$78^6 \to 4$	$8^{4n+2} \to 4$
$78^3 \to 2$	$78^7 \to 2$	Hence 78^{5562} will yield
$78^4 \to 6$	$78^8 \to 6$	four as the units digit

Similarly, $56^1 \to 6$, $56^2 \to 6$, $56^3 \to 6$ $\to 56^{256}$ will yield 6 as the units digit.

Similarly,

$97^1 \to 7$	$7^{4n+1} \to 7$
$97^2 \to 9$	$7^{4n+2} \to 9$
$97^3 \to 3$	Hence, 97^{1250} will yield a units digit of 9.
$97^4 \to 1$	

Hence, the required units digit is given by $4 \times 6 \times 9 \to$ **6** (answer).

Problem 1.21 Find the GCD and the LCM of the numbers P and Q where $P = 2^3 \times 5^3 \times 7^2$ and $Q = 3^3 \times 5^4$.

Solution GCD or HCF is given by the lowest powers of the common factors.

Thus, $\quad$ GCD = 5^3.

LCM is given by the highest powers of all factors available.

Thus, $\quad$ LCM = $2^3 \times 3^3 \times 5^4 \times 7^2$

Problem 1.22 A school has 378 girl students and 675 boy students. The school is divided into strictly boys or strictly girls sections. All sections in the school have the same number of students. Given this information, what are the number of sections in the school.

Solution The answer will be given by the HCF of 378 and 675.

$$378 = 2 \times 3^3 \times 7$$
$$675 = 3^3 \times 5^2$$

Hence, HCF of the two is $3^3 = 27$.

Hence, the number of sections is given by: $\frac{378}{27} + \frac{675}{27} = 14 + 25 =$ **39 sections.**

Level of Difficulty (LOD) I

1. The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be
   (a) 0 (b) 9 (c) 7 (d) 2
   (e) 8
2. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the changed number.
   (a) 28 (b) 19 (c) 37 (d) 46
   (e) 82
3. When we multiply a certain two-digit number by the sum of its digits, 405 is achieved. If you multiply the number written in reverse order of the same digits by the sum of the digits, we get 486. Find the number.
   (a) 81 (b) 45
   (c) 36 (d) 54
   (e) None of these
4. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.
   (a) 11, 4 (b) 12, 3
   (c) 13, 2 (d) 10, 5
   (e) 9, 6
5. The difference between two numbers is 48 and the difference between the arithmetic mean and the geometric mean is two more than half of 1/3 of 96. Find the numbers.
   (a) 49, 1 (b) 12, 60
   (c) 50, 2 (d) 36, 84
   (e) None of these
6. If $A381$ is divisible by 11, find the value of the smallest natural number A?
   (a) 5 (b) 6
   (c) 7 (d) 9
   (e) None of these
7. If $381A$ is divisible by 9, find the value of smallest natural number A?
   (a) 5 (b) 5
   (c) 7 (d) 9
   (e) 6
8. What will be the remainder obtained when $(9^6 + 1)$ will be divided by 8?
   (a) 0 (b) 3 (c) 7 (d) 2
   (e) 1
9. Find the ratio between the LCM and HCF of 5, 15 and 20?
   (a) 8 : 1 (b) 14 : 3 (c) 12 : 2 (d) 12 : 1
   (e) 1 : 12
10. Find the LCM of 5/2, 8/9, 11/14.
    (a) 280 (b) 360
    (c) 420 (d) 220
    (e) None of these
11. If the number A is even, which of the following will be true?
    (a) 3A will always be divisible by 6
    (b) 3A + 5 will always be divisible by 11
    (c) $(A^2 + 3)/4$ will be divisible by 7
    (d) All of these
    (e) None of these
12. A five-digit number is taken. Sum of the first four digits (excluding the number at the units digit) equals sum of all the five digits. Which of the following will not divide this number necessarily?
    (a) 10 (b) 2 (c) 4 (d) 5
    (e) None of these
13. A number $15B$ is divisible by 6. Which of these will be true about the positive integer B?
    (a) B will be even
    (b) B will be odd
    (c) B will be divisble by 6
    (d) Both (a) and (c)
    (e) None of these
14. Two numbers $P = 2^3.3^{10}.5$ and $Q = 2^5.3^1.7^1$ are given. Find the GCD of P and Q.
    (a) 2.3.5.7 (b) 3.2^2
    (c) $2^2.3^2$ (d) $2^3.3$
    (e) $2^3 \times 3^{10} \times 5^1 \times 7^1$
15. Find the units digit of the expression $25^{6251} + 36^{528} + 73^{54}$.
    (a) 4 (b) 0 (c) 6 (d) 5
    (e) 1
16. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$.
    (a) 4 (b) 0 (c) 6 (d) 5
    (e) 8

17. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (c) 7 (d) 0
(e) 8

18. Find the units digit of the expression $11^1.12^2.13^3.14^4.15^5.16^6$.
(a) 4 (b) 3
(c) 7 (d) 0
(e) None of these

19. Find the number of zeroes at the end of 1090!
(a) 270 (b) 268
(c) 269 (d) 271
(e) None of these

20. If 146! is divisible by 5^n, then find the maximum value of n.
(a) 34 (b) 35
(c) 36 (d) 37
(e) None of these

21. Find the number of divisors of 1420.
(a) 14 (b) 15
(c) 13 (d) 12
(e) None of these

22. Find the HCF and LCM of the polynomials $(x^2 - 5x + 6)$ and $(x^2 - 7x + 10)$.
(a) $(x - 2)$, $(x - 2)(x - 3)(x - 5)$
(b) $(x - 2)$, $(x - 2)(x - 3)$
(c) $(x - 3)$, $(x - 2)(x - 3)(x - 5)$
(d) $(x - 2)$, $(x - 2)(x - 3)(x - 5)^2$
(e) None of these

Directions for Questions 23–25: Given two different prime numbers P and Q, find the number of divisors of the following:

23. $P.Q$
(a) 2 (b) 4 (c) 6 (d) 8
(e) 5

24. P^2Q
(a) 2 (b) 4 (c) 6 (d) 8
(e) 9

25. P^3Q^2
(a) 2 (b) 4 (c) 6 (d) 12
(e) 8

26. The sides of a pentagonal field (not regular) are 1737 metres, 2160 metres, 2358 metres, 1422 metres and 2214 metres respectively. Find the greatest length of the tape by which the five sides may be measured completely?
(a) 7 (b) 13 (c) 11 (d) 9
(e) 10

27. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. Find the total number of sections thus formed?
(a) 24 (b) 32
(c) 16 (d) 20
(e) None of these

28. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing?
(a) 34 (b) 46 (c) 26 (d) 44
(e) 45

29. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?
(a) 9997 (b) 9793
(c) 9895 (d) 9487
(e) 9897

30. Find the least number that when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.
(a) 364 (b) 2254
(c) 2964 (d) 3234
(e) 2884

31. Four bells ring at the intervals of 6, 8, 12 and 18 seconds. They start ringing together at 12'O' clock. After how many seconds will they ring together again?
(a) 72 (b) 84 (c) 60 (d) 48
(e) 144

32. For question 31 find how many times will they ring together during the next 12 minutes? (including the 12 minute mark)
(a) 9 (b) 10
(c) 11 (d) 12
(e) None of these

33. The units digit of the expression $125^{813} \times 553^{3703} \times 4532^{828}$ is
(a) 4 (b) 2
(c) 0 (d) 5
(e) None of these

34. Which of the following is not a perfect square?
(a) 1,00,856 (b) 3,25,137
(c) 9,45,729 (d) All of these
(e) None of these

35. Which of the following can never be in the ending of a perfect square?
(a) 6 (b) 00 (c) 000 (d) 1
(e) 9

36. The LCM of 5, 8,12, 20 will not be a multiple of
(a) 3 (b) 9
(c) 8 (d) 5
(e) None of these

37. Find the number of divisors of 720 (including 1 and 720).
(a) 25 (b) 28 (c) 29 (d) 30
(e) 32

38. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
(a) $(x-3)(x+3)(4-x^2)$
(b) $4(4-x^2)(x+3)$
(c) $(4-x^2)(x-3)$
(d) $(4-x)(x-3)$
(e) None of these

39. GCD of $x^2 - 4$ and $x^2 + x - 6$ is
(a) $x+2$ (b) $x-2$
(c) x^2-2 (d) x^2+2
(e) None of these

40. The number A is not divisible by 3. Which of the following will not be divisible by 3?
(a) $9 \times A$ (b) $2 \times A$
(c) $18 \times A$ (d) $24 \times A$
(e) None of these

41. Find the remainder when the number 9^{100} is divided by 8?
(a) 1 (b) 2 (c) 0 (d) 4
(e) 2

42. Find the remainder of 2^{1000} when divided by 3?
(a) 1 (b) 2 (c) 4 (d) 6
(e) 0

43. Decompose the number 20 into two terms such that their product is the greatest.
(a) $x_1 = x_2 = 10$ (b) $x_1 = 5, x_2 = 15$
(c) $x_1 = 16, x_2 = 4$ (d) $x_1 = 8, x_2 = 12$
(e) None of these

44. Find the number of zeroes at the end of 50!
(a) 13 (b) 11 (c) 5 (d) 12
(e) 10

45. Which of the following can be a number divisible by 24?
(a) 4,32,15,604 (b) 25,61,284
(c) 13,62,480 (d) All of these
(e) None of these

46. For a number to be divisible by 88, it should be
(a) Divisible by 22 and 8
(b) Divisible by 11 and 8
(c) Divisible by 11 and thrice by 2
(d) Both (b) and (c)
(e) All of these

47. Find the number of divisors of 10800.
(a) 57 (b) 60
(c) 72 (d) 64
(e) None of these

48. Find the GCD of the polynomials $(x+3)^2(x-2)(x+1)^2$ and $(x+1)^3(x+3)(x+4)$.
(a) $(x+3)^3(x+1)^2(x-2)(x+4)$
(b) $(x+3)(x-2)(x+1)(x+4)$
(c) $(x+3)(x+1)^2$
(d) $(x+1)(x+3)^2$
(e) None of these

49. Find the LCM of $(x+3)(6x^2+5x+4)$ and $(2x^2+7x+3)(x+3)$
(a) $(2x+1)(x+3)(3x+4)$
(b) $(4x^2-1)(x+3)^2(3x+4)$
(c) $(4x^2-1)(x+3)(3x+4)$
(d) $(2x-1)(x+3)(3x+4)$
(e) None of these

50. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by
(a) 12 (b) 24
(c) 6 (d) All of these
(e) None of these

51. Some birds settled on the branches of a tree. First, they sat one to a branch and there was one bird too many. Next they sat two to a branch and there was one branch too many. How many branches were there?
(a) 3 (b) 4 (c) 5 (d) 6
(e) 2

52. The square of a number greater than 1000 that is not divisible by three, when divided by three, leaves a remainder of
(a) 1 always (b) 2 always
(c) 0 (d) either 1 or 2
(e) Cannot be said

53. The value of the expression $(15^3 \cdot 21^2)/(35^2 \cdot 3^4)$ is
(a) 3 (b) 15 (c) 21 (d) 12
(e) 35

54. If $A = \left(\frac{-3}{4}\right)^3, B = \left(\frac{-2}{5}\right)^2, C = (0.3)^2, D = (-1.2)^2$ then
(a) $A > B > C > D$ (b) $D > A > B > C$
(c) $D > B > C > A$ (d) $D > C > A > B$
(e) None of these

55. If $2 < x < 4$ and $1 < y < 3$, then find the ratio of the upper limit for $x + y$ and the lower limit of $x - y$.
(a) 6 (b) 7
(c) 8 (d) 4
(e) None of these

56. The sum of the squares of the digits constituting a positive two-digit number is 13. If we subtract 9 from that number, we shall get a number written by the same digits in the reverse order. Find the number?
(a) 12 (b) 32 (c) 42 (d) 52
(e) 23

57. The product of a natural number by the number written by the same digits in the reverse order is 2430. Find the numbers.
(a) 54 and 45 (b) 56 and 65
(c) 53 and 35 (d) 85 and 58
(e) None of these

58. Find two natural numbers whose difference is 66 and the least common multiple is 360.
(a) 120 and 54 (b) 90 and 24
(c) 180 and 114 (d) 130 and 64
(e) None of these

59. Find the pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13.
(a) 58 and 13 or 16 and 29
(b) 68 and 23 or 36 and 49
(c) 18 and 73 or 56 and 93
(d) 78 and 13 or 26 and 39
(e) None of these

60. Find two natural numbers whose sum is 85 and the least common multiple is 102.
(a) 30 and 55 (b) 17 and 68
(c) 35 and 55 (d) 51 and 34
(e) None of these

61. Find the pairs of natural numbers the difference of whose squares is 55.
(a) 28 and 27 or 8 and 3
(b) 18 and 17 or 18 and 13
(c) 8 and 27 or 8 and 33
(d) 9 and 18 or 8 and 27
(e) None of these

62. Which of these is greater.
(a) 54^4 or 21^{12} (b) $(0.4)^4$ or $(0.8)^3$

63. Is it possible for a common fraction whose numerator is less than the denominator to be equal to a fraction whose numerator is greater than the denominator?
(a) Yes (b) No

64. What digits should be put in place of c in $38c$ to make it divisible by
(1) 2 (2) 3 (3) 4 (4) 5
(5) 6 (6) 9 (7) 10

65. Find the LCM and HCF of the following numbers: (54, 81, 135 and 189), (156, 195) and (1950, 5670 and 3900)

66. The last digit in the expansions of the three digit number $(34x)^{43}$ and $(34x)^{44}$ are 7 and 1 respectively. What can be said about the value of x?
(a) $x = 5$ (b) $x = 3$
(c) $x = 6$ (d) $x = 2$

Directions for Questions 67–68: Amitesh buys a pen, a pencil and an eraser for Rs. 41. If the least cost of any of the three items is Rs. 12 and it is known that a pen costs less than a pencil and an eraser costs more than a pencil, answer the following questions:

67. What is the cost of the pen?
(a) 12 (b) 13
(c) 14 (d) 15

68. If it is known that the eraser's cost is not divisible by 4, the cost of the pencil could be:
(a) 12 (b) 13
(c) 14 (d) 15

69. A naughty boy Amrit watches a Sachin Tendulkar innings and acts according to the number of runs he sees Sachin scoring. The details of these are given below.

1 run	Place a orange in the basket
2 runs	Place a mango in the basket
3 runs	Place a pear in the basket
4 runs	Remove a pear and a mango from the basket

One fine day, at the start of the match, the basket is empty. The sequence of runs scored by Sachin in that innings are given as 11232411234232341121314. At the end of the above innings, how many more oranges were there compared to mangoes inside the basket? (The Basket was empty initially).

(a) 4 (b) 5
(c) 6 (d) 7

70. In the famous Bel Air Apartments in Ranchi, there are three watchmen meant to protect the precious fruits in the campus. However, one day a thief got in without being noticed and stole some precious mangoes. On the way out however, he was confronted by the three watchmen, the first two of whom asked him to part with $1/3^{rd}$ of the fruits and one more. The last asked him to part with $1/5^{th}$ of the mangoes and 4 more. As a result he had no mangoes left. What was the number of mangoes he had stolen?
(a) 12 (b) 13
(c) 15 (d) None of these

71. A hundred and twenty digit number is formed by writing the first x natural numbers in front of each other as 12345678910111213... Find the remainder when this number is divided by 8.
(a) 6 (b) 7
(c) 2 (d) 0

72. A test has 80 questions. There is one mark for a correct answer, while there is a negative penalty of $-1/2$ for a wrong answer and $-1/4$ for an unattempted question. What is the number of questions answered correctly, if the student has scored a net total of 34.5 marks.
(a) 45 (b) 48
(c) 54 (d) Cannot be determined

73. For the question 72, if it is known that he has left 10 questions unanswered, the number of correct answers are:
(a) 45 (b) 48
(c) 54 (d) Cannot be determined

74. Three mangoes, four guavas and five watermelons cost Rs. 750. Ten watermelons, six mangoes and 9 guavas cost Rs.1580. What is the cost of six mangoes, ten watermelons and 4 guavas?
(a) 1280 (b) 1180
(c) 1080 (d) Cannot be determined

75. From a number M subtract 1. Take the reciprocal of the result to get the value of 'N'. Then which of the following is necessarily true?
(a) $M^N \le 2$ (b) $M^N > 3$
(c) $1 < M^N < 3$ (d) $1 < M^N < 5$

76. The cost of four mangoes, six guavas and sixteen watermelons is Rs. 500, while the cost of seven magoes, nine guavas and nineteen watermelons is Rs. 620. What is the cost of one mango, one guava and one watermelon?
(a) 120 (b) 40
(c) 150 (d) Cannot be determined

77. For the question above, what is the cost of a mango?
(a) 20 (b) 14
(c) 15 (d) Cannot be determined

78. The following is known about three real numbers, x, y and z.
$-4 \le x \le 4$, $-8 \le y \le 2$ and $-8 \le z \le 2$. Then the range of values that $M = xz/y$ can take is best represented by:
(a) $-\infty < x < \infty$ (b) $-16 \le x \le 8$
(c) $-8 \le x \le 8$ (d) $-16 \le x \le 16$

79. A man sold 38 pieces of clothing (combined in the form of shirts, trousers and ties). If he sold at least 11 pieces of each item and he sold more shirts than trousers and more trousers than ties, then the number of ties that he must have sold is:
(a) Exactly 11 (b) At least 11
(c) At least 12 (d) Cannot be determined

80. For the question 79, find the number of shirts he must have sold?
(a) At least 13 (b) At least 14
(c) At least 15 (d) At most 16.

81. Find the least number which when divided by 12, 15, 18 or 20 leaves in each case a remainder 4.
(a) 124 (b) 364
(c) 184 (d) None of these

82. What is the least number by which 2800 should be multiplied so that the product may be a perfect square?
(a) 2 (b) 7
(c) 14 (d) None of these

83. The least number of 4 digits which is a perfect square is:
(a) 1064 (b) 1040
(c) 1024 (d) 1012

84. The least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18 is
(a) 94 (b) 184
(c) 364 (d) 74

85. What is the least 3 digit number that when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?
(a) 131 (b) 161
(c) 121 (d) None of these

86. The highest common factor of 70 and 245 is equal to
(a) 35 (b) 45
(c) 55 (d) 65

87. Find the least number, which must be subtracted from 7147 to make it a perfect square.
(a) 86 (b) 89
(c) 91 (d) 93

88. Find the least square number which is divisible by 6, 8 and 15
(a) 2500 (b) 3600
(c) 4900 (d) 4500

89. Find the least number by which 30492 must be multiplied or divided so as to make it aperfect square.
(a) 11 (b) 7
(c) 3 (d) 2

90. The greatest 4-digit number exactly divisible by 88 is
(a) 8888 (b) 9768
(c) 9944 (d) 9988

91. By how much is three fourth of 116 greater than four fifth of 45?
(a) 31 (b) 41
(c) 46 (d) None of these

92. If 5625 plants are to be arranged in such a way that there are as many rows as there are plants in a row, the number of rows will be.
(a) 95 (b) 85
(c) 65 (d) None of these

93. A boy took a seven digit number ending in 9 and raised it to an even power greater than 2000. He then took the number 17 and raised it to a power which leaves the remainder 1 when divided by 4. If he now multiples both the numbers, what will be the unit's digit of the number he so obtains?
(a) 7 (b) 9
(c) 3 (d) Cannot be determined

94. Two friends were discussing their marks in an examination. While doing so they realized that both the numbers had the same prime factors, although Raveesh got a score which had two more factors than Harish. If their marks are represented by one of the options as given below, which of the following options would correctly represent the number of marks they got.
(a) 30,60 (b) 20,80
(c) 40,80 (d) 20,60

95. A number is such that when divided by 4, 5, 6, or 7 it leaves the remainder 2, 3, 4, or 5 respectively. Which is the largest number below 4000 that satisfies this property?
(a) 3358 (b) 3988
(c) 3778 (d) 2938

Level of Difficulty (LOD) II

1. The arithmetic mean of two numbers is smaller by 24 than the larger of the two numbers and the GM of the same numbers exceeds by 12 the smaller of the numbers. Find the numbers.
(a) 6 and 54 (b) 8 and 56
(c) 12 and 60 (d) 7 and 55
(e) None of these

2. Find the number of numbers between 200 and 300, both included, which are not divisible by 2, 3, 4 and 5.
(a) 27 (b) 26
(c) 25 (d) 28
(e) None of these

3. Given x and n are integers, $(15n^3 + 6n^2 + 5n + x)/n$ is not an integer for what condition?
(a) n is positive
(b) x is divisible by n
(c) x is not divisible by n
(d) (a) and (c)
(e) None of these

4. The unit digit in the expression $36^{234}*33^{512}*39^{180} - 54^{29}*25^{123}*31^{512}$ will be
(a) 8 (b) 0 (c) 6 (d) 5
(e) 4

5. The difference of $10^{25} - 7$ and $10^{24} + x$ is divisible by 3 for $x = ?$
(a) 3 (b) 2 (c) 4 (d) 6
(e) 7

6. Find the value of x in $\sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{3x}}}} = x$.
(a) 1 (b) 3 (c) 6 (d) 12
(e) 9

7. If a number is multiplied by 22 and the same number is added to it, then we get a number that is half the square of that number. Find the number
(a) 45 (b) 46
(c) 47 (d) data insufficient
(e) None of these

8. $12^{55}/3^{11} + 8^{48}/16^{18}$ will give the digit at units place as
(a) 4 (b) 6 (c) 8 (d) 0
(e) 5

9. The mean of $1, 2, 2^2 .... 2^{31}$ lies in between

(a) 2^{24} to 2^{25} (b) 2^{25} to 2^{26}
(c) 2^{26} to 2^{27} (d) 2^{29} to 2^{30}
(e) 2^{23} to 2^{24}

10. xy is a number that is divided by ab where $xy < ab$ and gives a result $0.xyxyxy...$ then ab equals
(a) 11 (b) 33 (c) 99 (d) 66
(e) 88

11. A number xy is multiplied by another number ab and the result comes as pqr, where $r = 2y$, $q = 2(x + y)$ and $p = 2x$ where $x, y < 5$, $q \neq 0$. The value of ab may be:
(a) 11 (b) 13
(c) 31 (d) 22
(e) None of these

12. $[x]$ denotes the greatest integer value just below x and $\{x\}$ its fractional value. The sum of $[x]^3$ and $\{x\}^2$ is -7.91. Find x.
(a) -2.03 (b) -1.97
(c) -2.97 (d) -1.7
(e) None of these

13. $16^5 + 2^{15}$ is divisible by
(a) 31 (b) 13 (c) 27 (d) 33
(e) 12

14. If $AB + XY = 1XP$, where $A \neq 0$ and all the letters signify different digits from 0 to 9, then the value of A is:
(a) 6 (b) 7
(c) 9 (d) 8
(e) Any value above 6

Directions for Questions number 15–16: Find the possible integral values of x.

15. $|x - 3| + 2|x + 1| = 4$
(a) 1 (b) -1
(c) 3 (d) 2
(e) There are many solutions

16. $x^2 + |x - 1| = 1$
(a) 1 (b) -1
(c) 0 (d) 1 or 0
(e) -2

17. If $4^{n+1} + x$ and $4^{2n} - x$ are divisible by 5, n being an even integer, find the least value of x.
(a) 1 (b) 2
(c) 3 (d) 0
(e) None of these

18. If the sum of the numbers $(a25)^2$ and a^3 is divisible by 9, then which of the following may be a value for a ?
(a) 1 (b) 7
(c) 9 (d) 8
(e) There is no value

19. If $|x - 4| + |y - 4| = 4$, then how many integer values can the set (x, y) have?
(a) Infinite (b) 5
(c) 16 (d) 9
(e) 25

20. $[3^{32}/50]$ gives remainder and $\{.\}$ denotes the fractional part of that. The fractional part is of the form $(0 \cdot bx)$. The value of x could be
(a) 2 (b) 4
(c) 6 (d) 8
(e) None of these

21. The sum of two numbers is 20 and their geometric mean is 20% lower than their arithmetic mean. Find the ratio of the numbers.
(a) 4 : 1 (b) 9 : 1
(c) 1 : 1 (d) 17 : 3
(e) 5 : 1

22. The highest power on 990 that will exactly divide 1090! is
(a) 101 (b) 100 (c) 108 (d) 109
(e) 110

23. If 146! is divisible by 6^n, then find the maximum value of n.
(a) 74 (b) 70 (c) 76 (d) 75
(e) 73

24. The last two digits in the multiplication of $35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$ is
(a) 00 (b) 40
(c) 30 (d) 10
(e) None of these

25. The expression $333^{555} + 555^{333}$ is divisible by
(a) 2 (b) 3
(c) 37 (d) 111
(e) All of these

26. $[x]$ denotes the greatest integer value just below x and $\{x\}$ its fractional value. The sum of $[x]^2$ and $\{x\}^1$ is 25.16. Find x.
(a) 5.16 (b) -4.84
(c) Both (a) and (b) (d) 4.84
(e) Cannot be determined

27. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.
(a) 18 (b) 39 (c) 49 (d) 28
(e) 30

28. Find two numbers such that their sum, their product and the differences of their squares are equal.
(a) $\left(\frac{3+\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$ or $\left(\frac{3+\sqrt{2}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$
(b) $\left(\frac{3+\sqrt{7}}{2}\right)$ and $\left(\frac{1+\sqrt{7}}{2}\right)$ or $\left(\frac{3+\sqrt{6}}{2}\right)$ and $\left(\frac{1-\sqrt{6}}{2}\right)$
(c) $\left(\frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{1-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}\right)$ and $\left(\frac{1+\sqrt{5}}{2}\right)$
(d) All of these
(e) None of these

29. The sum of the digits of a three-digit number is 17, and the sum of the squares of its digits is 109. If we subtract 495 from that number, we shall get a number consisting of the same digits written in the reverse order. Find the number.
(a) 773 (b) 863
(c) 683 (d) 944
(e) 684

30. Find the number of zeros in the product: $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots\ldots 98^{98} \times 99^{99} \times 100^{100}$
(a) 1200 (b) 1300
(c) 1050 (d) 1225
(e) None of these

31. Find the pairs of a natural number whose greatest common divisor is 5 and the least common multiple is 105.
(a) 5 and 105 or 15 and 35
(b) 6 and 105 or 16 and 35
(c) 5 and 15 or 15 and 135
(d) 5 and 20 or 15 and 35
(e) None of these

32. The denominator of an irreducible fraction is greater than the numerator by 2. If we reduce the numerator of the reciprocal fraction by 3 and subtract the given fraction from the resulting one, we get 1/15. Find the given fraction.
(a) $\frac{2}{4}$ (b) $\frac{3}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{9}$
(e) $\frac{8}{9}$

33. A two-digit number exceeds by 19 the sum of the squares of its digits and by 44 the double product of its digits. Find the number.
(a) 72 (b) 62 (c) 22 (d) 12
(e) 15

34. The sum of the squares of the digits constituting a two-digit positive number is 2.5 times as large as the sum of its digits and is larger by unity than the trebled product of its digits. Find the number.
(a) 13 and 31 (b) 12 and 21
(c) 22 and 33 (d) 14 and 41
(e) None of these

35. The units digit of a two-digit number is greater than its tens digit by 2, and the product of that number by the sum of its digits is 144. Find the number.
(a) 14 (b) 24 (c) 46 (d) 35
(e) 20

36. Find the number of zeroes in the product: $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$
(a) 8 (b) 9 (c) 12 (d) 13
(e) 10

37. The power of 45 that will exactly divide 123! is
(a) 28 (b) 30 (c) 31 (d) 59
(e) 29

38. Three numbers are such that the second is as much lesser than the third as the first is lesser than the second. If the product of the two smaller numbers is 85 and the product of two larger numbers is 115 find the middle number.
(a) 9 (b) 8 (c) 12 (d) 30
(e) 10

39. Find the smallest natural number *n* such that *n*! is divisible by 990.
(a) 3 (b) 5

(c) 11 (d) 12
(e) None of these

40. $\sqrt{x}\sqrt{y} = \sqrt{xy}$ is true only when
(a) $x > 0, y > 0$ (b) $x > 0$ and $y < 0$
(c) $x < 0$ and $y > 0$ (d) All of these
(e) None of these

Directions for Questions 41–60: Read the instructions below and solve the questions based on this.

In an examination situation, always solve the following type of questions by substituting the given options, to arrive at the solution.

However, as you can see, there are no options given in the questions here since these are meant to be an exercise in equation writing (which I believe is a primary skill required to do well in aptitude exams testing mathematical aptitude). Indeed, if these questions had options for them, they would be rated as LOD 1 questions. But since the option-based solution technique is removed here, I have placed these in the LOD 2 category.

41. Find the two-digit number that meets the following criteria. If the number in the units place exceeds, the number in its tens by 2 and the product of the required number with the sum of its digits is equal to 144.
42. The product of the digits of a two-digit number is twice as large as the sum of its digits. If we subtract 27 from the required number, we get a number consisting of the same digits written in the reverse order. Find the number?
43. The product of the digits of a two-digit number is one-third that number. If we add 18 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number?
44. The sum of the squares of the digits of a two-digit number is 13. If we subtract 9 from that number, we get a number consisting of the same digits written in the reverse order. Find the number?
45. A two-digit number is thrice as large as the sum of its digits, and the square of that sum is equal to the trebled required number. Find the number?
46. Find a two-digit number that exceeds by 12 the sum of the squares of its digits and by 16 the doubled product of its digits.
47. The sum of the squares of the digits constituting a two-digit number is 10, and the product of the required number by the number consisting of the same digits written in the reverse order is 403. Find the 2 numbers that satisfy these conditions?
48. If we divide a two-digit number by the sum of its digits, we get 4 as a quotient and 3 as a remainder. Now, if we divide that two-digit number by the product of its digits, we get 3 as a quotient and 5 as a remainder. Find the two-digit number.
49. There is a natural number that becomes equal to the square of a natural number when 100 is added to it, and to the square of another natural number when 169 is added to it. Find the number?
50. Find two natural numbers whose sum is 85 and whose least common multiple is 102.
51. Find two-three digit numbers whose sum is a multiple of 504 and the quotient is a multiple of 6.
52. The difference between the digits in a two-digit number is equal to 2, and the sum of the squares of the same digits is 52. Find all the possible numbers?
53. If we divide a given two-digit number by the product of its digits, we obtain 3 as a quotient and 9 as a remainder. If we subtract the product of the digits constituting the number, from the square of the sum of its digits, we obtain the given number. Find the number.
54. Find the three-digit number if it is known that the sum of its digits is 17 and the sum of the squares of its digits is 109. If we subtract 495 from this number, we obtain a number consisting of the same digits written in reverse order.
55. The sum of the cubes of the digits constituting a two-digit number is 243 and the product of the sum of its digits by the product of its digits is 162. Find the two two-digit number?
56. The difference between two numbers is 16. What can be said about the total numbers divisible by 7 that can lie in between these two numbers.
57. Arrange the following in descending order:
111^4, 110.109.108.107, 109.110.112.113
58. If $3 \leq x \leq 5$ and $4 \leq y \leq 7$. Find the greatest value of xy and the least value of x/y.
59. Which of these is greater:
(a) 200^{300} or 300^{200} or 400^{150}
(b) 5^{100} and 2^{200}
(c) 10^{20} and 40^{10}
60. The sum of the two numbers is equal to 15 and their arithmetic mean is 25 per cent greater than its geometric mean. Find the numbers.
61. Define a number K such that it is the sum of the squares of the first M natural numbers.(i.e. $K = 1^2 + 2^2 + \ldots + M^2$)

where $M < 55$. How many values of M exist such that K is divisible by 4?

(a) 10 (b) 11
(c) 12 (d) None of these

62. M is a two digit number which has the property that: The product of factorials of it's digits > sum of factorials of its digits
How many values of M exist?
(a) 56 (b) 64
(c) 63 (d) None of these

63. A natural number when increased by 50% has its number of factors unchanged. However, when the value of the number is reduced by 75%, the number of factors is reduced by 66.66%. One such number could be:
(a) 32 (b) 84
(c) 126 (d) None of these

64. Find the 28383rd term of the series: 123456789101112....
(a) 3 (b) 4
(c) 9 (d) 7

65. If you form a subset of integers chosen from between 1 to 3000, such that no two integers add up to a multiple of nine, what can be the maximum number of elements in the subset. (Include both 1 and 3000.)
(a) 1668 (b) 1332
(c) 1333 (d) 1336

66. The series of numbers (1,1/2,1/3,1/4 ....1/1972) is taken. Now two numbers are taken from this series (the first two) say x, y. Then the operation $x + y + x.y$ is performed to get a consolidated number. The process is repeated. What will be the value of the set after all the numbers are consolidated into one number.
(a) 1970 (b) 1971
(c) 1972 (d) None of these

67. K is a three digit number such that the ratio of the number to the sum of its digits is least? What is the difference between the hundreds and the tens digits of K?
(a) 9 (b) 8
(c) 7 (d) None of these

68. In the question 67, what can be said about the difference between the tens and the units digit?
(a) 0 (b) 1
(c) 2 (d) None of these

69. For the above question, for how many values of K will the ratio be the highest?
(a) 9 (b) 8
(c) 7 (d) None of these

70. A triangular number is defined as a number which has the property of being expressed as a sum of consecutive natural numbers starting with 1. How many triangular numbers less than 1000, have the property that they are the difference of squares of two consecutive natural numbers?
(a) 20 (b) 21
(c) 22 (d) 23

71. x and y are two positive integers. Then what will be the sum of the coefficients of the expansion of the expression $(x + y)^{44}$? Answer: 2^{44}
(a) 2^{43} (b) $2^{43} + 1$
(c) 2^{44} (d) $2^{44} - 1$

72. What is the remainder when $9 + 9^2 + 9^3 + \ldots 9^{2n+1}$ is divided by 6?
(a) 1 (b) 2
(c) 3 (d) 4

73. The remainder when the number 123456789101112 ......484950 is divided by 16 is:
(a) 3 (b) 4
(c) 5 (d) 6

74. What is the highest power of 3 available in the expression $58! - 38!$
(a) 17 (b) 18
(c) 19 (d) None of these

75. Find the remainder when the number represented by 22334 raised to the power $(1^2 + 2^2 + .. + 66^2)$ is divided by 5?
(a) 2 (b) 4
(c) 0 (d) None of these

76. What is the total number of divisors of the number $12^{33} \times 34^{23} \times 2^{47}$?
(a) 4658. (b) 9316
(c) 2744 (d) None of these

77. For the question 76, which of the following will represent the sum of factors of the number (such that only odd factors are counted)?
(a) $\frac{(3^{34}-1)}{2} \times \frac{(17^{24}-1)}{16}$ (b) $(3^{34}-1) \times (17^{24}-1)$
(c) $\frac{(3^{34}-1)}{33}$ (d) None of these

78. What is the remainder when $(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \ldots (1152!)^3$ is divided by 1152?
(a) 125 (b) 225
(c) 325 (d) 205

79. A set S is formed by including some of the first One thousand natural numbers. S contains the maximum

number of numbers such that they satisfy the following conditions:

1. No number of the set S is prime.
2. When the numbers of the set S are selected two at a time, we always see co prime numbers

What is the number of elements in the set S?

(a) 11 (b) 12
(c) 13 (d) 7

Find the last two digits of the following numbers

80. $101 \times 102 \times 103 \times 197 \times 198 \times 199$
(a) 54 (b) 74
(c) 64 (d) 84

81. $65 \times 29 \times 37 \times 63 \times 71 \times 87$
(a) 05 (b) 95
(c) 15 (d) 25

82. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 85$
(a) 25 (b) 35
(c) 75 (d) 85

83. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 62$
(a) 70 (b) 30
(c) 10 (d) 90

84. $75 \times 35 \times 47 \times 63 \times 71 \times 87 \times 82$
(a) 50 (b) 70
(c) 30 (d) 90

85. $(201 \times 202 \times 203 \times 204 \times 246 \times 247 \times 248 \times 249)^2$
(a) 36 (b) 56
(c) 76 (d) 16

86. Find the remainder when 7^{99} is divided by 2400.
(a) 1 (b) 343
(c) 49 (d) 7

87. Find the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3.
(a) 729 (b) 1000
(c) 752 (d) 1

88. Arun, Bikas and Chetakar have a total of 80 coins among them. Arun triples the number of coins with the others by giving them some coins from his own collection. Next, Bikas repeats the same process. After this Bikas now has 20 coins. Find the number of coins he had at the beginning?
(a) 22 (b) 20
(c) 18 (d) 24

89. The super computer at Ram Mohan Roy Seminary takes an input of a number N and a X where X is a factor of the number N. In a particular case N is equal to $83p$ $796161q$ and X is equal to 11 where $0 < p < q$, find the sum of remainders when N is divided by $(p + q)$ and p successively.
(a) 6 (b) 3
(c) 2 (d) 9

90. On March 1st 2016, Sherry saved Re.1. Everyday starting from March 2rd 2016, he saved Re.1 more than the previous day. Find the first date after March 1st 2016 at the end of which his total savings will be a perfect square.
(a) 17th March 2016 (b) 18th April 2016
(c) 26th March 2016 (d) None of these

91. What is the rightmost digit preceding the zeroes in the value of 20^{53}?
(a) 2 (b) 8
(c) 1 (d) 4

92. What is the remainder when 2(8!) – 21(6!) divides 14(7!) + 14(13!)?
(a) 1 (b) 7!
(c) 8! (d) 9!

93. How many integer values of x and y are there such that $4x + 7y = 3$, while $|x| < 500$ and $|y| < 500$?
(a) 144 (b) 141
(c) 143 (d) 142

94. If $n = 1 + m$, where m is the product of four consecutive positive integers, then which of the following is/are true?
(A) n is odd (B) n is not a multiple of 3
(C) n is a perfect square
(a) All three (b) A and B only
(c) A and C only (d) None of these

95. How many two-digit numbers less than or equal to 50, have the product of the factorials of their digits less than or equal to the sum of the factorials of their digits?
(a) 18 (b) 16
(c) 15 (d) None of these

96. A candidate takes a test and attempts all the 100 questions in it. While any correct answer fetches 1 mark, wrong answers are penalised as follows; one-tenth of the questions carry 1/10 negative mark each, one-fifth of the questions carry 1/5 negative marks each and the rest of the questions carry ½ negative mark each. Unattempted questions carry no marks. What is the difference between the maximum and the minimum marks that he can score?
(a) 100 (b) 120
(c) 140 (d) None of these

Directions for Questions 97 to 99: A mock test is taken at AMS Learning Systems. The test paper comprises of questions in three levels of difficulty—LOD1, LOD2 and LOD 3.

The following table gives the details of the positive and negative marks attached to each question type:

Difficulty level	*Positive marks for answering the question correctly*	*Negative marks for answering the question wrongly*
LOD 1	4	2
LOD 2	3	1.5
LOD 3	2	1

The test had 200 questions with 80 on LOD 1 and 60 each on LOD 2 and LOD 3.

97. If a student has solved 100 questions exactly and scored 120 marks, the maximum number of incorrect questions that he/she might have marked is:
(a) 44 (b) 56
(c) 60 (d) None of these

98. If Amit attempted the least number of questions and got a total of 130 marks, and if it is known that he attempted at least one of every type, then the number of questions he must have attempted is:
(a) 34 (b) 35
(c) 36 (d) None of these

99. In the above question, what is the least number of questions he might have got incorrect?
(a) 0 (b) 1
(c) 2 (d) None of these

100. Amitabh has a certain number of toffees, such that if he distributes them amongst ten children he has nine left, if he distributes amongst 9 children he would have 8 left, if he distributes amongst 8 children he would have 7 left ... and so on until if he distributes amongst 5 children he should have 4 left. What is the second lowest number of toffees he could have with him?
(a) 2519 (b) 7559
(c) 8249 (d) 5039

Level of Difficulty (LOD) III

1. What two-digit number is less than the sum of the square of its digits by 11 and exceeds their doubled product by 5?
(a) 15, 95 (b) 95
(c) Both (a) and (b) (d) 15, 95 and 12345
(e) None of these

2. Find the lower of the two successive natural numbers if the square of the sum of those numbers exceeds the sum of their squares by 112.
(a) 6 (b) 7 (c) 8 (d) 9
(e) 10

3. First we increased the denominator of a positive fraction by 3 and then we decreased it by 5. The sum of the resulting fractions proves to be equal to 2/3. Find the denominator of the fraction if its numerator is 2.
(a) 7 (b) 8 (c) 12 (d) 9
(e) 13

4. Find the last two digits of: $15 \times 37 \times 63 \times 51 \times 97 \times 17$.
(a) 35 (b) 45 (c) 55 (d) 85
(e) 75

5. Let us consider a fraction whose denominator is smaller than the square of the numerator by unity. If we add 2 to the numerator and the denominator, the fraction will exceed 1/3. If we subtract 3 from the numerator and the denominator, the fraction will be positive but smaller than 1/10. Find the value?
(a) $\frac{3}{8}$ (b) $\frac{4}{15}$
(c) $\frac{5}{24}$ (d) $\frac{6}{35}$
(e) None of these

6. Find the sum of all three-digit numbers that give a remainder of 4 when they are divided by 5.
(a) 98,270 (b) 99,270
(c) 1,02,090 (d) 90,270
(e) None of these

7. Find the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7.
(a) 686 (b) 676
(c) 666 (d) 656
(e) None of these

8. Find the sum of all odd three-digit numbers that are divisible by 5.
(a) 50,500 (b) 50,250
(c) 50,000 (d) 49,500
(e) 51,250

9. The product of a two-digit number by a number consisting of the same digits written in the reverse order is equal to 2430. Find the lower number.
(a) 54 (b) 52 (c) 63 (d) 65
(e) 45

10. Find the lowest of three numbers as described: If the cube of the first number exceeds their product by 2, the cube of the second number is smaller than their product by 3, and the cube of the third number exceeds their product by 3.

(a) $3^{1/3}$ (b) $9^{1/3}$
(c) 2 (d) Any of these
(e) None of these

11. How many pairs of natural numbers are there the difference of whose squares is 45.

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

12. Find all two-digit numbers such that the sum of the digits constituting the number is not less than 7; the sum of the squares of the digits is not greater than 30; the number consisting of the same digits written in the reverse order is not larger than half the given number.

(a) 52 (b) 51 (c) 49 (d) 53
(e) 50

13. In a four-digit number, the sum of the digits in the thousands, hundreds and tens is equal to 14, and the sum of the digits in the units, tens and hundreds is equal to 15. Among all the numbers satisfying these conditions, find the number the sum of the squares of whose digits is the greatest.

(a) 2572 (b) 1863
(c) 2573 (d) 1858
(e) None of these

14. In a four-digit number, the sum of the digits in the thousands and tens is equal to 4, the sum of the digits in the hundreds and the units is 15, and the digit of the units exceeds by 7 the digit of the thousands. Among all the numbers satisfying these conditions, find the number the sum of the product of whose digit of the thousands by the digit of the units and the product of the digit of the hundreds by that of the tens assumes the least value.

(a) 4708 (b) 1738
(c) 2629 (d) 1812
(e) None of these

15. If we divide a two-digit number by a number consisting of the same digits written in the reverse order, we get 4 as a quotient and 15 as a remainder. If we subtract 1 from the given number, we get the sum of the squares of the digits constituting that number. Find the number?

(a) 71 (b) 83
(c) 99 (d) 86
(e) None of these

16. Find the two-digit number the quotient of whose division by the product of its digits is equal to 8/3, and the difference between the required number and the number consisting of the same digits written in the reverse order is 18

(a) 86 (b) 42
(c) 75 (d) 12
(e) None of these

17. Find the two-digit number if it is known that the ratio of the required number and the sum of its digits is 8 as also the quotient of the product of its digits and that of the sum is 14/9.

(a) 54 (b) 72
(c) 27 (d) 45
(e) None of these

18. If we divide the unknown two-digit number by the number consisting of the same digits written in the reverse order, we get 4 as a quotient and 3 as a remainder. If we divide the required number by the sum of its digits, we get 8 as a quotient and 7 as a remainder. Find the number?

(a) 81 (b) 91
(c) 71 (d) 72
(e) None of these

19. The last two-digits in the multiplication $122 \times 123 \times 125 \times 127 \times 129$ will be

(a) 20 (b) 50 (c) 30 (d) 40
(e) 60

20. The remainder obtained when $43^{101} + 23^{101}$ is divided by 66 is:

(a) 2 (b) 10 (c) 5 (d) 0
(e) 35

21. The last three-digits of the multiplication 12345×54321 will be

(a) 865 (b) 745
(c) 845 (d) 945
(e) 875

22. The sum of the digits of a three-digit number is 12. If we subtract 495 from the number consisting of the same digits written in reverse order, we shall get the required number. Find that three-digit number if the sum of all pairwise products of the digits constituting that number is 41.

(a) 156 (b) 237

(c) 197 (d) 159
(e) Both (a) and (b)

23. A three-digit positive integer *abc* is such that $a^2 + b^2 + c^2 = 74$. a is equal to the doubled sum of the digits in the tens and units places. Find the number if it is known that the difference between that number and the number written by the same digits in the reverse order is 495.
(a) 813 (b) 349
(c) 613 (d) 713
(e) None of these

24. Represent the number 1.25 as a product of three positive factors so that the product of the first factor by the square of the second is equal to 5 if we have to get the lowest possible sum of the three factors.
(a) $x_1 = 2.25, x_2 = 5, x_3 = 0.2$
(b) $x_1 = 1.25, x_2 = 4, x_3 = 4.5$
(c) $x_1 = 1.25, x_2 = 2, x_3 = 0.5$
(d) $x_1 = 1.25, x_2 = 4, x_3 = 2$
(e) None of these

25. Find a number x such that the sum of that number and its square is the least.
(a) −0.5 (b) 0.5 (c) −1.5 (d) 1.5
(e) 1

26. When $2222^{5555} + 5555^{2222}$ is divided by 7, the remainder is
(a) 0 (b) 2 (c) 4 (d) 5

27. If x is a number of five-digits which when divided by 8, 12, 15 and 20 leaves respectively 5, 9, 12 and 17 as remainders, then find x such that it is the lowest such number?
(a) 10017 (b) 10057 (c) 10097 (d) 10137
(e) None of these

28. $3^{2n} - 1$ is divisible by 2^{n+3} for n =
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these

29. $10^n - \left(5+\sqrt{17}\right)^n$ is divisible by 2^{n+2} for what whole number value of n?
(a) 2 (b) 3 (c) 7 (d) 8
(e) None of these

30. $\dfrac{32^{32^{32}}}{9}$ will leave a remainder:
(a) 4 (b) 7 (c) 1 (d) 2

31. Find the remainder that the number $1989 \cdot 1990 \cdot 1992^3$ gives when divided by 7;
(a) 0 (b) 1 (c) 5 (d) 2
(e) 3

32. Find the remainder of 2^{100} when divided by 3.
(a) 3 (b) 0 (c) 1 (d) 2
(e) None of these

33. Find the remainder when the number 3^{1989} is divided by 7.
(a) 1 (b) 5 (c) 6 (d) 4
(e) 3

34. Find the last digit of the number $1^2 + 2^2 + ... + 99^2$.
(a) 0 (b) 1 (c) 2 (d) 3
(e) 5

35. Find gcd $(2^{100} - 1, 2^{120} - 1)$.
(a) $2^{20} - 1$ (b) $2^{40} - 1$
(c) $2^{60} - 1$ (d) $2^{10} - 1$
(e) $2^{40} - 1$

36. Find the gcd (111...11 hundred ones ; 11...11 sixty ones).
(a) 111...forty ones (b) 111...twenty five ones
(c) 111...twenty ones (d) 111...sixty ones
(e) None of these

37. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 ... + 99^3$.
(a) 0 (b) 1 (c) 2 (d) 5
(e) 8

38. Find the GCD of the numbers $2n + 13$ and $n + 7$.
(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

39. $\dfrac{32^{32^{32}}}{7}$
(a) 4 (b) 2 (c) 1 (d) 3
(e) 5

40. The remainder when $10^{10} + 10^{100} + 10^{1000} + ... + 10^{10000000000}$ is divided by 7 is
(a) 0 (b) 1 (c) 2 (d) 5
(e) 4

41. n is a number, such that $2n$ has 28 factors and $3n$ has 30 factors. $6n$ has?
(a) 35 (b) 32
(c) 28 (d) None of these

42. Suppose the sum of n consecutive integers is $x + (x + 1) + (x + 2) + (x + 3) + ... + (x + (n - 1)) = 1000$,

then which of the following cannot be true about the number of terms n

(a) The number of terms can be 16

(b) The number of terms can be 5

(c) The number of terms can be 25

(d) The number of terms can be 20

43. The remainder when $2^2 + 22^2 + 222^2 + 2222^2 + \ldots\ldots.(222\ldots..49 \text{ twos})^2$ is divided by 9 is:

(a) 2 (b) 5

(c) 6 (d) 7

44. $N = 202 \times 20002 \times 200000002 \times 20000000000000002 \times 200000000\ldots.2$ (31 zeroes) The sum of digits in this multiplication will be:

(a) 112 (b) 160

(c) 144 (d) Cannot be determined

45. Twenty five sets of problems on Data Interpretation–one each for the DI sections of 25 CATALYST tests were prepared by the AMS research team. The DI section of each CATALYST contained 50 questions of which exactly 35 questions were unique, i.e. they had not been used in the DI section of any of the other 24 CATALYSTs. What could be the maximum possible number of questions prepared for the DI sections of all the 25 CATALYSTs put together?

(a) 1100 (b) 975

(c) 1070 (d) 1055

46. In the above question, what could be the minimum possible number of questions prepared?

(a) 890 (b) 875

(c) 975 (d) None of these

Directions for Questions 47–49: At a particular time in the twenty first century there were seven bowlers in the Indian cricket team's list of 16 players short listed to play the next world cup. Statisticians discovered that that if you looked at the number of wickets taken by any of the 7 bowlers of the current Indian cricket team, the number of wickets taken by them had a strange property. The numbers were such that for any team selection of 11 players (having 1 to 7 bowlers) by using the number of wickets taken by each bowler and attaching coefficients of +1, 0, or –1 to each value available and adding the resultant values, any number from 1 to 1093, both included could be formed. If we denote W_1, W_2, W_3, W_4, W_5, W_6 and W_7 as the 7 values in the ascending order what could be the answer to the following questions:

47. Find the value of $W_1 + 2W_2 + 3W_3 + 4W_4 + 5W_5 + 6W_6$.

(a) 2005 (b) 1995

(c) 1985 (d) None of these

48. Find the index of the largest power of 3 contained in the product $W_1 W_2 W_3 W_4 W_5 W_6 W_7$.

(a) 15 (b) 10

(c) 21 (d) 6

49. If the sum of the seven coefficients is 0, find the smallest number that can be obtained.

(a) – 1067 (b) – 729

(c) – 1040 (d) – 1053

Directions for Questions 50 and 51: Answer these questions on the basis of the information given below.

In the ancient game of Honololo the task involves solving a puzzle asked by the chief of the tribe. Anybody answering the puzzle correctly is given the hand of the most beautiful maiden of the tribe. Unfortunately, for the youth of the tribe, solving the puzzle is not a cakewalk since the chief is the greatest mathematician of the tribe.

In one such competition the chief called everyone to attention and announced openly:

"A three-digit number '*mnp*' is a perfect square and the number of factors it has is also a perfect square. It is also known that the digits m, n and p are all distinct. Now answer my questions and win the maiden's hand."

50. If $(m + n + p)$ is also a perfect square, what is the number of factors of the six-digit number *mnpmnp*?

(a) 32 (b) 72

(c) 48 (d) Cannot be determined

51. If the fourth power of the product of the digits of the number *mnp* is not divisible by 5, what is the number of factors of the nine-digit number, *mnpmnpmnp*?

(a) 32 (b) 72

(c) 48 (d) Cannot be determined

52. In a cricket tournament organised by the ICC, a total of 15 teams participated. Australia, as usual won the tournament by scoring the maximum number of points. The tournament is organised as a single round robin tournament—where each team plays with every other team exactly once. 3 points are awarded for a win, 2 points are awarded for a tie/washed out match and 1 point is awarded for a loss. Zimbabwe had the lowest score (in terms of points) at the end of the tournament. Zimbabwe scored a total of 21 points. All the 15 national teams got a distinct score (in terms of points scored). It is also known that at least one match played by the Australian team was tied/washed out. Which of the following is always true for the Australian team?

(a) It had at least two ties/washouts.

(b) It had a maximum of 3 losses.

(c) It had a maximum of 9 wins.
(d) All of the above.

53. What is the remainder when 128^{1000} is divided by 153
(a) 103 (b) 145 (c) 118 (d) 52

54. Find the remainder when $50^{51^{52}}$ is divided by 11.
(a) 6 (b) 4 (c) 7 (d) 3

55. Find the remainder when $32^{33^{34}}$ is divided by 11.
(a) 5 (b) 4 (c) 10 (d) 1

56. Find the remainder when $30^{72^{87}}$ is divided by 11.
(a) 5 (b) 9 (c) 6 (d) 3

57. Find the remainder when $50^{56^{52}}$ is divided by 11.
(a) 7 (b) 5 (c) 9 (d) 10

58. Find the remainder when $33^{34^{35}}$ is divided by 7.
(a) 5 (b) 4 (c) 6 (d) 2

59. Let S_m denote the sum of the squares of the first m natural numbers. For how many values of $m < 100$, is S_m a multiple of 4?
(a) 50 (b) 25 (c) 36 (d) 24

60. For the above question, for how many values will the sum of cubes of the first m natural numbers be a multiple of 5 (if $m < 50$)?
(a) 20 (b) 21
(c) 22 (d) None of these

61. How many integer values of x and y satisfy the expression $4x + 7y = 3$ where $|x| < 1000$ and $|y| < 1000$.
(a) 284 (b) 285
(c) 286 (d) None of these

Hints and Solutions I

3. The two numbers should be factors of 405. A factor search will yield the factors. (look only for 2 digit factors of 405 with sum of digits between 1 to 19).
Also $405 = 5 \times 3^4$. Hence: 15×27
45×9 are the only two options.
From these factors pairs only the second pair gives us the desired result.
i.e. Number × sum of digits = 405.
Hence, the answer is 45.

5. Two more than half of 1/3rd of 96 = 18. Also since we are given that the difference between the AM and GM is 18, it means that the GM must be an integer. From amongst the options, only option 1 gives us a GM which is an integer. Thus, checking for option 1, we get the GM=7 and AM=18.

8. 9^6 when divided by 8, would give a remainder of 1. Hence, the required answer would be 2.

12. The essence of this question is in the fact that the last digit of the number is 0. Naturally, the number is necessarily divisible by 2,5 and 10. Only 4 does not necessarily divide it.

15. The units digit would be given by 5 + 6 + 9 (numbers ending in 5 and 6 would always end in 5 and 6 irrespective of the power and 3^{54} will give a units digit equivalent to 3^{4n+2} which would give us a unit digit of 3^2 i.e.9)

20. The number of 5's in 146! can be got by [146/5] + [29/5] + [5/5]= 29+5+1 = 35

26. The sides of the pentagon being 1422, 1737, 2160, 2214 and 2358, the least difference between any two numbers is 54. Hence, the correct answer will be a factor of 54.
Further, since there are some odd numbers in the list, the answer should be an odd factor of 54.
Hence, check with 27, 9 and 3 in that order. You will get 9 as the HCF.

29. The LCM of the 4 numbers is 612. The highest 4 digit number which would be a common multiple of all these 4 numbers is 9792. Hence, the correct answer is 9793.

40. If A is not divisible by 3, it is obvious that 2A would also not be divisible by 3, as 2A would have no '3' in it.

46. Any number divisible by 88, has to be necessarily divisible by 11, 2, 4, 8, 44 and 22. Thus, each of the first three options is correct.

50. Three consecutive natural numbers, starting with an even number would always have at least three 2's as their prime factors and also would have at least one multiple of 3 in them. Thus, 6, 12 and 24 would each divide the product.

51. When the birds sat one on a branch, there was one extra bird. When they sat 2 to a branch one branch was extra.
To find the number of branches, go through options.
Checking option (a)
If three were 3 branches, there would be 4 birds. (this would leave one bird without branch as per the question.)
When 4 birds would sit 2 to a branch there would be 1 branch free (as per the question). Hence, the answer (a) is correct.

52. The number would either be $(3n + 1)^2$ or $(3n + 2)^2$. In the expansion of each of these the only term which would not be divisible by 3 would be the square of 1 and 2 respectively. When divided by 3, both of these give 1 as remainder.
56. For the sum of squares of digits to be 13, it is obvious that the digits should be 2 and 3. So the number can only be 23 or 32. Further, the number being referred to has to be 32 since the reduction of 9, reverses the digits.
60. Solve using options. Option (d) 51 and 34 satisfies the required conditions.
62. (a) $21^{12} = (21^3)^4$
    Since $21^3 > 54$, $21^{12} > 54^4$.
    (b) $10.4)^4 = (4/10)^4 = 1024/10000 = 0.1024$.
    $(0.8)^3 = (8/10)^3 = 512/1000 = 0.512$
    Hence, $(0.8)^3 > (0.4)^4$.

Solutions for 67 & 68:

The given condition says that Pen<Pencil<Eraser. Also, since the least cost of the three is Rs.12, if we allocate a minimum of 12 to each we use up 36 out of the 41 available. The remaining 5 can be distributed as 0,1,4 or 0,2 and 3 giving possible values of Case 1: 12,13 and 16 or Case 2: 12,14 and 15.

67. In both cases, the cost of the pen is 12.
68. If the cost of the eraser is not divisible by 4, it means that Case 2 holds true. For this case, the cost of the pencil is 14.
71. The last 3 digits of the number would determine the remainder when it is divided by 8. The number upto the 120th digit would be 1234567891011… 646. 646 divided by 8 gives us a remainder of 6.
76. $(7M + 9G + 19W) - (4M + 6G + 16W) = 120$. Hence, $1M + 1G + 1W = 40$
77. The cost of a mango cannot be uniquely determined here because we have only 2 equations between 3 variables, and there is no way to eliminate one variable.
82. $2800 = 20 \times 20 \times 7$. Thus, we need to multiply or divide with 7 in order to make it a perfect square.
92. The correct arrangement would be 75 plants in a row and 75 rows since 5625 is the square of 75.
95. In order to solve this question you need to realize that remainders of 2, 3, 4 and 5 in the case of 4, 5, 6 and 7 respectively, means remainders of –2 in each case. In order to find the number which leaves remainder –2 when divided by these numbers you need to first find the LCM of 4, 5, 6 and 7 and subtract 2 from them. Since the LCM is 210, the first such number which satisfies this condition is 208. However, the question has asked us to find the largest such number below 4000. So you need to look at multiples of the LCM and subtract 2. The required number is 3990 – 2 = 3988

Hints and Solutions II

1. If a and b are two numbers, then their $AM = (a + b)/2$ and $GM = (ab)^{0.5}$. Use the options to answer the question.
2. Use the principal of counting given in the theory of the chapter. Start with 101 numbers and reduce all the numbers which are divisible with 2, 3 and 5. Ensure that there is no double counting in this process.
4. Find the units digits individually and subtract.
5. Suppose you were to solve the same question for $10^3 - 7$ and $10^2 + x$.
   $10^3 - 7 = 993$ and $10^2 + x = 100 + x$.
   Difference $= 993 - x$
   For $10^4 - 7$ and $10^3 + x$
   The difference would be $9993 - (1000 + x)$
   $= (8993 - x)$
   For $10^5 - 7$ and $10^3 + x$
   Difference: $99993 - (10000 + x) = 89993 - x$
   You should realize that the difference for the given question would be $8999 \ldots\ldots 9\,3 - x$. For this difference to be divisible by 3, x must be 2 (since that is the only option which will give you a sum of digits divisible by 3.)
6. The value of x should be such that the left hand side after completely removing the square root signs should be an integer. For this to happen, first of all the square root of $3x$ should be an integer. Only 3 and 12 from the options satisfy this requirement. If we try to put x as 12, we get the square root of $3x$ as 6. Then the next point at which we need to remove the square root sign would be $12+2(6) = 24$ whose square root would be an irrational number. This leaves us with only 1 possible value ($x = 3$). Checking for this value of x we can see that the expression is satisfied as LHS = RHS.
7. Solve using options

8. $12^{55}/3^{11} = 3^{44}.4^{55} \rightarrow 4$ as units place.
   Similarly, $8^{48}/16^{18} = 2^{72} \rightarrow 6$ as the units place.
   Hence, 0 is the answer.
9. $1 + 2 + 2^2 + \ldots + 2^{31} = 2^{32} - 1$

   Hence, the average will be: $\dfrac{2^{32}-1}{32} = 2^{27} - 1/2^5$
   which lies between 2^{26} and 2^{27}.
   Hence the answer will be (c).
10. The denominator 99 has the property that the decimals it gives rise to are of the form 0.*xyxyxy*. This question is based on this property of 99. Option *c* is correct.
11. The value of *b* has to be 2 since, $r = 2y$. Hence, option *d* is the only choice.
12. For $[x]^3 + \{x\}^2$ to give -7.91,
    $[x]^3$ should give -8 (hence, $[x]$ should be -2]
    Further, $\{x\}^2$ should be $+0.09$.
    Both these conditions are satisfied by -1.7.
    Hence option (d) is correct.
13. $16^5 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5 + 1) \rightarrow$ Hence, is divisible by 33.
14. For $A + X$ to be X, the only possible situation is that the value of a should be either 0 or 9.

15&16. Use the method of physical counting of all possible values of x.

15. $|x - 3| + 2|x + 1| = 4$ can happen under three broad conditions.
    (a) When $2|x + 1| = 0$, then $|x - 3|$ should be equal to 4.
    Putting $x = -1$, both these conditions are satisfied.
    (b) When $2|x + 1| = 2$, x should be 0, then $|x - 3|$ should also be 2. This does not happen.
    (c) When $2|x + 1| = 4$, x should be $+1$ or -3, in either case $|x - 3|$ which should be zero does not give the desired value.
17. 4^{n+1} represents an odd power of 4 (and hence would end in 4). Similarly, 4^{2n} represents an even power of 4 (and hence would end in 6). Thus, the least number 'x' that would make both $4^{n+1} + x$ and $4^{2n} - x$ divisible by 5 would be for $x = 1$.
18. Check through options.
19. The two modulus values have to add up to 4 together. This can happen by (0 + 4), (1 + 3), (2 + 2), (3 + 1) or (4 + 0). Find the individual values of x and y for which these set of values are satisfied.
20. The numerator of $3^{32}/50$ would be a number that would end in 1. Consequently, the decimal of the form .*bx* would always give us a value of x as 2.
21. Use standard formulae for Am and GM.
23. For finding the highest power of 6 that divides 146!, we need to get the number of 3's that would divide 146!. The same can be got by: [146/3] + [48/3] + [16/3] + [5/3] = 70.
25. Both 333^{555} and 555^{333} are divisible by 3,37 and 111. Further, the sum of the two would be an even number and hence divisible by 2. Thus, all the four options divide the given number.

26–32. Solve through options.

30. The number of zeroes will equal the number of fives. To count the number of fives extract all terms having 5 as a prime factor and add the number of fives they give you. You should get the following two A.P.S. whose sum will give you the answer 5, 10, 15 .... 100 and 25, 50, 75, 100.
33. Use trial and error to solve by going through the options. 72 fits both the conditions perfectly.

34–35. Solve through options.

36. The number of zeroes depends on the number of 5's and the number of 2's, whichever is less. Here, the constraint is the number of 2's and not 5's (the usual case).
37. Check the powers of 5 and 3^2 contained in 123! The lower value amongst these will be the answer.
38. Solve through options.
39. Find the largest prime factor contained in 990 and check its factorial value for divisibility by 990.
40. Check for different positive and negative values of x and y according to the options.
41. Since, we are not given options here we should go ahead by looking within the factors of 144 (especially the two digit ones)
    The relevant factors are 72,48, 36, 24, 18 and 12. Thinking this way creates an option for you where there is none available and from this list of numbers you can easily identify 24 as the required answer.

42–46. Write simple equations for each of the questions and solve.

47. Since the sum of squares of the digits of the two digit number is 10, the only possibility of the numbers are 31 and 13.
48. Write simple equations for each of the questions and solve.
49. We can see from the description that the number (say X) must be such that $X + 100$ and $X + 169$ both must be perfect squares. Thus we are looking for two perfect squares which are 69 apart from each other. This

would happen for 34^2 and 35^2 since their difference would be $(35 - 34)(35 + 34) = 69$

50–56. Write simple equations for each of the questions and solve.

57. Between 111^4, $110 \times 109 \times 108 \times 107$, $109 \times 110 \times 112 \times 113$.

It can be easily seen that

$111 \times 111 \times 111 \times 111 > 110 \times 109 \times 108 \times 107$

also $109 \times 110 \times 112 \times 113 > 109 \times 110 \times 108 \times 107$

Further the product $111 \times 111 \times 111 \times 111 > 109 \times 110 \times 112 \times 113$ (since, the sum of the parts of the product are equal on the LHS and the RHS and the numbers on the LHS are closer to each other than the numbers on the RHS.

58. Both x and y should be highest for xy to be maximum. Similarly x should be minimum and y should be maximum for x/y to be minimum.

59. $200^{300} = (200^6)^{50}$

$300^{200} = (300^4)^{50}$

$400^{150} = (400^3)^{50}$

Hence 200^{300} is greater.

Resolve to a common base for comparing

63. You need to solve this question using trial and error.

For 32 (option 1):

$32 = 2^5$. Hence 6 factors. On increasing by 50%, $48 = 2^4 \times 3^1$ has 10 factors. Thus the number of factors is increasing when the number is increased by 50% which is not what the question is defining for the number. Hence, 32 is not the correct answer.

Checking for option (b) 84.

$84 = 2^2 \times 3^1 \times 7^1 \rightarrow (2+1)(1+1)(1+1) = 12$ factors

On increasing by 50% $\rightarrow 126 = 2^1 \times 32 \times 7^1 \rightarrow (1+1)(2+1)(1+1) = 12$ factors. (no change in number of factors).

Second Condition: When the value of the number is reduced by 75% $\rightarrow$ 84 would become 21 $(3^1 \times 7^1)$ and the number of factors would be $2 \times 2 = 4$ – a reduction of 66.66% in the number of factors.

65. In order to solve this question, think of the numbers grouped in groups of 9 as:

{1, 2, 3, 4, 5, 6, 7, 8, 9} {10, 11, 12.....18} and so on till {2989, 2990...2997} – A total of 333 complete sets. From each set we can take 4 numbers giving us a total of $333 \times 4 = 1332$ numbers.

Apart from this, we can also take exactly 1 multiple of 9 (any one) and also the last 3 numbers viz 2998, 2999 and 3000. Thus, there would be a total of $1332 + 4 = 1336$ numbers.

70. Basically every odd triangular number would have this property, that it is the difference of squares of two consecutive natural numbers. Thus, we need to find the number of triangular numbers that are odd.

3, 15, 21, 45, 55, 91, 105, 153, 171, 231, 253, 325, 351, 435, 465, 561, 595, 703, 741, 861, 903 – A total of 21 numbers.

73. The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16 (because 10000 is a multiple of 16. This principle is very similar in logic to why we look at last 2 digits for divisibility by 4 and the last 3 digits for divisibility by 8). Thus, the required answer would be the remainder of 4950/16 which is 6.

78. $1152 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^7 \times 3^2$. Essentially every number starting from $4!^3$ would be divisible perfectly by 1152 since each number after that would have at least 7 twos and 2 threes.

Thus, the required remainder is got by the first three terms:

$(1 + 8 + 216)/1152 = 225/1152$ gives us 225 as the required remainder.

88. Bikas's movement in terms of the number of coins would be:

$B \rightarrow 3B$ (when Arun triples everyone's coins) $\rightarrow B$.

Think of this as: When Bikas triples everyone's coins, and is left with 20 it means that the other 3 have 60 coins after their coins are tripled. This means that before the tripling by Bikas, the other three must have had 20 coins—meaning Bikas must have had 60 coins. But $60 = 3B \rightarrow B = 20$.

92. $[7!\,(14 + 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8)]/[7!\,(16 - 3)] = [(14 + 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8)]/[(13)] \rightarrow$ remainder 1.

Hence, the original remainder must be 7! (because for the sake of simplification of the numbers in the question we have cut the 7! From the numerator and the denominator in the first step.

96. Product of factorials < Sum of factorials would occur for any number that has either 0 or 1 in it.

The required numbers uptil and including 50 are: 10 to 19, 20, 21, 30, 31, 40, 41, 50. Besides for the number 22, the product of factorials of the digits would be equal to the sum of factorials of the digits. Thus a total of 18 numbers.

100. The least number would be (LCM of 10, 9, 8, 7, 6 and $5 - 1) = 2519$. The second least number $= 2520 \times 2 - 1 = 5039$.

Hints and Solutions III

1–5. Solve through options.

6. Use AP with first term 104 and last term 999 and common difference 5.
7. Find the first 2 digit number which gives a remainder of 3 when divided by 7 and then find the largest such number (10 and 94 respectively). Use Arithmetic Progression formulae to add the numbers.
8. Use AP with first term 105 and last term 995 and common difference is 10.
10. The cubes of the numbers are $x - 3$, $x + 2$ and $x + 3$. Use options and you will see that (a) is the answer.
11. $(x^2 - y^2) = 45$. i.e. $(x - y)(x + y) = 45$. The factors of 45 possible are, 15, 3; 9, 5 and 45, 1.
    Hence, the numbers are 9 and 6, or 7 and 2 or 23 and 22.

12–18. Use options to check the given conditions.

19. The answer will be 50 since, 125*122 will give 50 as the last two digits.
20. The remainder theorem is to be used.
    $43^{101} + 23^{101} = (43 + 23)(.....)$
    Hence, when divided by 66 the remainder will be zero.
21. The unit's digit will be $1 \times 5 = 5$ (no carry over.) The tens digit will be (4*1 + 5*2) = 4 (carry over 1). The hundreds digit will be (3*1 + 4*2 + 5*1) = 6 + 1 (carried over) = 7. Hence, answer is 745.

22–25. Use options to solve.

26. Use the rule of indices and remainder theorem.
27. Options are not provided as it is an LOD 3 question. If they were there you should have used options.
28. Use trial and error
29. Use options.
30. Use remainder theorem and look at patterns by applying the rules of indices.
    We get the value as:

$$5 \xrightarrow{\frac{32.32.32..........32 \text{ times}}{9}}$$

$$7 \xrightarrow{\frac{32.32.32..........31 \text{ times}}{9}}$$

$$4 \xrightarrow{\frac{32.32.32..........30 \text{ times}}{9}}$$

$$7 \xrightarrow{\frac{32.32....29 \text{ times}}{9}}$$

$\rightarrow$ Looking at the pattern we will get 4 as the final remainder.

31. Use the remainder theorem and get the remainder as: $1 \times 2 \times 4 \times 4 \times 4/7 = 128/7 \rightarrow 2$ is the remainder. 32.
32. $2^{100}/3 = (2^4)^{25}/3 \rightarrow 1$.
33. Use the remainder theorem and try finding the patterns.
34. Find the last digit of the number got by adding $1^2 + 2^2 + \ldots 9^2$ (you will get 5 here). Then multiply by 10 to get zero as the answer.
35. $(2^{100} - 1)$ and $(2^{120} - 1)$ will yield the GCD as $2^{20} - 1$. (This has been explained in the theory of GCDs).
36. The GCDs of 100 ones and 60 ones will be twenty ones because 20 is the GCD of sixty and Hundred.
39. $\frac{32^{32^{32}}}{7} \rightarrow \frac{4^{32^{32}}}{7}$
    But $4^3/7$ gives us a remainder of 1.
    Hence we need to convert $4^{32^{32}}$ into 4^{3n+x} (Think why!!)
    Here again, we will be more interested in finding the value of x rather than n, since the remainder only depends on the value of x.

    [Concept Note: When we start to write as the remainder.
    $4^{32^{32}}$ in the form $4^3 \times 4^3 \times 4^3$ ...... n times $\times 4^x$ we are not bothered about how many times we can write 4^3 since it will continuously give us 1 every time as the remainder.

ANSWER KEY

Number System LOD I

1. a	36. b	65. LCM → 17010 HCF → 27 LCM → 780 HCF → 29 LCM → 245700 HCF → 30
2. a	37. d	66. b
3. b	38. e	67. a
4. b	39. b	68. b
5. a	40. b	69. c
6. c	41. a	70. c
7. e	42. a	71. a
8. d	43. a	72. d
9. d	44. d	73. b
10. e	45. c	74. b
11. a	46. e	75. a
12. c	47. b	76. b
13. d	48. c	77. d
14. d	49. c	78. a
15. b	50. d	79. a
16. c	51. a	80. b
17. b	52. a	81. c
18. d	53. b	82. b
19. a	54. c	83. c
20. b	55. e	84. c
21. d	56. b	85. c
22. a	57. a	86. a
23. b	58. b	87. c
24. c	59. d	88. b
25. d	60. d	89. b
26. d	61. a	90. c
27. c	62. a → $(21)^{12}$ b → $(0.8)^{3}$	91. d
28. d	63. b	92. d
29. b	64. 1. → 0, 2, 4, 6, 8 2. → 1, 4, 7 3. → 0, 4, 8 4. → 0, 5 5. → 4 6. → 7 7. → 0	93. a
30. e		94. c
31. a		95. c
32. b		
33. c		
34. d		
35. c		

Number System LOD II

1. a
2. b
3. c
4. c
5. b
6. b
7. b
8. d
9. c
10. c
11. d
12. d
13. d
14. c
15. b
16. d
17. a
18. e
19. c
20. a
21. a
22. c
23. b
24. a
25. e
26. c
27. e
28. e
29. b
30. b
31. a
32. b
33. a
34. a
35. b
36. b
37. a
38. e
39. c
40. a
41. 24
42. 63
43. 24
44. 32
45. 27
46. 64
47. 13, 31
48. 23
49. 1056
50. 51, 34
51. 144, 864
52. 46, 64
53. 63
54. 863
55. 36, 63
56. may be 2 or 3 depending upon the numbers
57. $111^4 > 109.110.112.113 > 110.109.108.107$
58. greatest → 35 least 3/7
59. (a) 200^{300} (b) 5^{100} (c) 10^{20}
60. 12, 3
61. c
62. c
63. b
64. a
65. d
66. c
67. b
68. a
69. a
70. b
71. c
72. c
73. d
74. a
75. b
76. d
77. a
78. b
79. b
80. e
81. b
82. c
83. d
84. a
85. c
86. b
87. d
88. b
89. d
90. d
91. a
92. b
93. c
94. a
95. a
96. c
97. b
98. a
99. a
100. d

Number System LOD III

1. a	33. c	
2. b	34. a	
3. d	35. a	
4. a	36. c	
5. b	37. a	
6. b	38. a	
7. b	39. a	
8. d	40. d	
9. e	41. a	
10. a	42. d	
11. c	43. c	
12. a	44. b	
13. e	45. d	
14. b	46. a	
15. e	47. a	
16. e	48. c	
17. b	49. c	
18. c	50. d	
19. b	51. d	
20. d	52. b	
21. b	53. d	
22. e	54. a	
23. a	55. c	
24. c	56. a	
25. a	57. b	
26. a	58. d	
27. d	59. d	
28. e	60. d	
29. e	61. b	
30. a		
31. d		
32. c		

2

PROGRESSIONS

The chapter on progressions essentially yields commonsense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems, since there is such a lot of commonality of logic between the problems associated with these two chapters. As already stated Block 1 of the six blocks of Chapters in QA accounts for anything between 12–18 marks in the CAT. This pattern has been consistently observed over the past decade.

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus each of the following series forms an arithmetic progression:

$$3, 7, 11, 15, \ldots$$
$$8, 2, -4, -10, \ldots$$
$$a, a + d, a + 2d, a + 3d, \ldots$$

The common difference is found by subtracting any term of the series from the next term.

That is, common difference of an $AP = (t_N - t_{N-1})$.

In the first of the above examples the common difference is 4; in the second it is −6; in the third it is d.

If we examine the series $a, a + d, a + 2d, a + 3d, \ldots$ we notice that *in any term the coefficient of d is always less by one than the position of that term in the series.*

Thus the rth term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If n be the number of terms, and if L denotes the last term or the nth term, we have

$$L = a + (n - 1)d$$

To Find the Sum of the Given Number of Terms in an Arithmetic Progression

Let a denote the first term d, the common difference, and n the total number of terms. Also, let L denote the last term, and S the required sum; then

$$S = \frac{n(a + L)}{2} \quad (1)$$

$$L = a + (n - 1)d \quad (2)$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \quad (3)$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus a is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in AP take $a - d$, a and $a + d$ as the three numbers as this reduces one unknown thereby making the solution easier.

To Find the Arithmetic Mean between any Two Given Quantities

Let a and b be two quantities and A be their arithmetic mean. Then since a, A, b, are in AP. We must have

$$b - A = A - a$$

Each being equal to the common difference;

This gives us $A = \frac{(a+b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in *AP*. The terms thus inserted are called the **arithmetic means.**

To Insert a Given Number of Arithmetic Means between Two Given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in *AP*, of which a is the first, and b is the last term.

Let d be the common difference;

then b = the $(n + 2)$th term $= a + (n + 1)d$

Hence, $d = \frac{(b-a)}{(n+1)}$

and the required means are

$$a + \frac{(b-a)}{n-1},\ a + \frac{2(b-a)}{n+1}, \ldots\ a + \frac{n(b-a)}{n+1}$$

Till now we have studied APs in their mathematical context. This was important for you to understand the basic mathematical construct of A.Ps. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis, (Especially under the time pressure that you are likely to face under the CAT and other aptitude exams). In such situations the mathematical processes for solving progressions based questions are likely to fail. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one:

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the A.P. 3, 7, 11.......

The conventional mathematical process for this question would involve using the formula.

$$T_n = a + (n - 1)\ d$$

Thus, for the 17th term we would do

$$T_{17} = 3 + (17 - 1) \times 4 = 3 + 16 \times 4 = 67$$

Most students would mechanically insert the values for a, n and d and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this:

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).

Similarly, in order to find the 37th term of the A.P. 3, 11 ..., All you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.

(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.)

2. Average of an A.P. and Corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = 2 + 6 + 10 + 14 + 18 + 22/6 = 12

Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P. (**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.)

We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (so that 1 + 6 = 7)
2nd and 5th(hence, 2 + 5 =7)
3rd and 4th (hence, 3 + 4 = 7)

Note that: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.

For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

3. Process for finding the sum of an A.P.

Once you can find a pair of corresponding terms for any A.P., you can easily find the sum of the A.P. by using the property of averages:

i.e. Sum = Number of terms × Average.

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a+(n-1)d)$.

4. Finding the common difference of an A.P., given 2 terms of an A.P.

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualize this A.P. as

–, –, 8, –, –, –, –, 28.

From the above figure, you can easily visualize that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

Illustration: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

Solution: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes 28 + 4 + 4 = 36.

Thus, the average of the A.P. = Average of 8th and 10th terms

= (28 + 36)/2 = 32.

Hence, the required answer is sum of the A.P. = 17 × 32 = 544.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an AP is –30, you should realize that 5 times the common difference will be equal to –30. (Since 12 – 7 = 5).

Hence, $d = -6$.

Note: Replace this algorithmic thinking in lieu of the mathematical thinking of:

$$12^{th}\text{ term} = a + 11d$$
$$7^{th}\text{ term} = a + 6d$$
$$\text{Hence, difference} = -30 = (a + 11d) - (a + 6d)$$
$$-30 = 5d$$
$$\therefore \quad d = -6.$$

5. Types of APs:Increasing and Decreasing A.P.s.

Depending on whether 'd' is positive or negative, an A.P. can be increasing or decreasing.

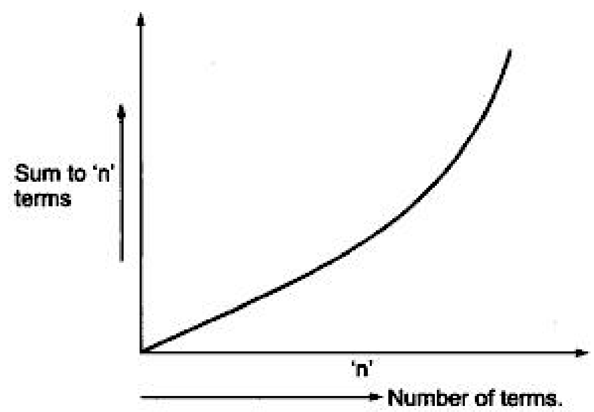
Let us explore these two types of A.P.s further:

(A) Increasing A.P.s:

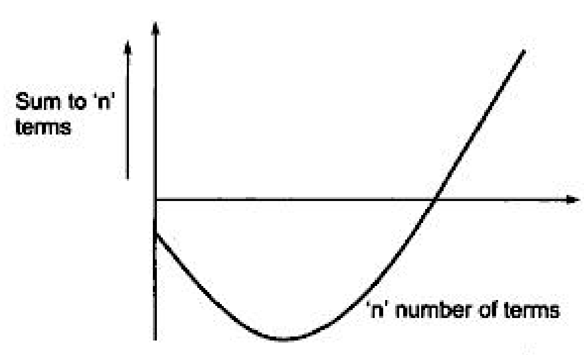
Every term of an increasing AP is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

Case 1: When the first term of the increasing A.P. is positive. In such a case the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below:



Case 2: When the first term of the increasing A.P. is negative. In such a case, the Sum of the A.P. plotted against the number of terms will give the following figure:



Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots\ldots.$$

(a) 27/14 (b) 21/13 (c) 49/27 (d) 256/147

Solution: Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series the values of the terms are:

First term = 1

Second term = 4/7 = 0.57

Third term = 9/63 = 0.14

Fourth term = 16/343 = 0.04

Fifth term = 25/2401 = 0.01

Addition upto the fifth term is approximately 1.76

Options 2 and 4 are smaller than 1.76 in value and hence cannot be correct.

That leaves us with options 1 and 3

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms)

Looking at the pattern we can predict that the sixth term will be

$$36/7^5 = 36/16807 = 0.002 \text{ (approx.)}$$

And the seventh term would be $49/7^6 = 49/117649 = 0.0004$ (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualized that the residual terms in the series are highly insignificant. Based on this judgement you realize that the answer will not reach 1.92 and will be restricted to 1.81. Hence the answer will be option 3.

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

USEFUL RESULTS

1. If the same quantity be added to, or subtracted from, all the terms of an AP, the resulting terms will form an *AP*, but with the same common difference as before.
2. If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
3. If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.
4. If $a, b, c, d, \ldots$ are in GP, they are also in continued proportion, since, by definition,

   $$a/b = b/c = c/d = \ldots = 1/r$$

   Conversely, a series of quantities in continued proportion may be represented by $x, xr, xr^2, \ldots$
5. If you have to assume 3 terms in *AP*, assume them as

   $$a - d, a, a + d \quad \text{or} \quad \text{as } a, a + d \text{ and } a + 2d$$

   For assuming 4 terms of an AP we use: $a - 3d$, $a - d$, $a + d$ and $a + 3d$

   For assuming 5 terms of an *AP*, take them as:

   $$a - 2d, a - d, a, a + d, a + 2d.$$

   These are the most convenient in terms of problems solving.
6. For assuming three terms of a GP assume them as

   $$a, ar \text{ and } ar^2 \quad \text{or} \quad \text{as } a/r, a \text{ and } ar$$
7. To find the sum of the first n natural numbers

   Let the sum be denoted by S; then

   $$S = 1 + 2 + 3 + \ldots + n, \text{ is given by}$$

   $$S = \frac{n(n+1)}{2}$$
8. To find the sum of the squares of the first n natural numbers

or $108 = -24n - 3n + 3n^2$ or $3n^2 - 27n - 108 = 0$

or $n^2 - 9n - 36 = 0$, or $n^2 - 12n + 3n - 36 = 0$

$n(n - 12) + 3(n - 12) = 0 \Rightarrow (n + 3)(n - 12) = 0$

The value of n (the number of terms) cannot be negative. Hence −3 is rejected.

So we have $n = 12$

Alternatively, we can directly add up individual terms and keep adding manually till we get a sum of 54. We will observe that this will occur after adding 12 terms. (In this case, as also in all cases where the number of terms is mentally manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation based process.)

Problem 2.10 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + …

(a) $n(n + 1)(n + 2)$
(b) $(n(n + 1)/12)(3n^2 + 19n + 26)$
(c) $((n + 1)(n + 2)(n + 3))/4$
(d) $(n^2 (n + 1)(n + 2)(n + 3))/3$

Solution In order to solve such problems in the examination, the option based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put $n = 1$, we should get the sum as 1.2.4 = 8. By substituting $n = 1$ in each of the four options we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6
Option (b) gives a value of: 8
Option (c) gives a value of: 6
Option (d) gives a value of: 8

From this check we can reject the options (a) and (c).

Now put $n = 2$. You can see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38

The correct option should also give 38 if we put $n = 2$ in the expression. Since, (a) and (c) have already been rejected, we only need to check for options (b) and (d).

Option (b) gives a value of 38
Option (d) gives a value of 80.
Hence, we can reject option (d) and get (b) as the answer.

***Note:* The above process is very effective for solving questions having options. The student should try to keep an eye open for the possibility of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).**

Level of Difficulty (LOD) I

1. How many terms are there in the AP 20, 25, 30,… 130.
   (a) 22 (b) 23 (c) 21 (d) 24
   (e) 25
2. Bobby was appointed to AMS Careers in the pay scale of Rs. 7000–500–12,500. Find how many years he will take to reach the maximum of the scale.
   (a) 11 years (b) 10 years
   (c) 9 years (d) 8 years
   (e) None of these
3. Find the 1st term of an AP whose 8th and 12th terms are respectively 39 and 59.
   (a) 5 (b) 6 (c) 4 (d) 3
   (e) 7
4. A number of squares are described whose perimetres are in GP. Then their sides will be in
   (a) AP (b) GP
   (c) HP (d) Nothing can be said
   (e) None of these
5. There is an *AP* 1, 3, 5…. Which term of this AP is 55?
   (a) 27th (b) 26th (c) 25th (d) 28th
   (e) 29th
6. How many terms are identical in the two *AP*s 1, 3, 5,… up to 120 terms and 3, 6, 9,… up to 80 terms?
   (a) 38 (b) 39 (c) 40 (d) 41
   (e) 42
7. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.
   (a) 5 (b) 10 (c) 15 (d) 20
   (e) (a), (b) & (c)
8. Find the 15th term of the sequence 20, 15, 10, …
   (a) −45 (b) −55 (c) −50 (d) 0
   (e) −45
9. A sum of money kept in a bank amounts to Rs. 1240 in 4 years and Rs. 1600 in 10 years at simple Interest. Find the sum.

116. Find the seventh term of the progression if its fifth term is known to be exactly divisible by 14.

(a) 36 (b) 40 (c) 43 (d) 22
(e) 24

26. A and B set out to meet each other from two places 165 km apart. A travels 15 km the first day, 14 km the second day, 13 km the third day and so on. B travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days (b) 5 days
(c) 6 days (d) 7 days
(e) 9 days

27. If a man saves Rs. 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) Rs. 21,478 (b) Rs. 21,578
(c) Rs. 22,578 (d) Rs. 22,478
(e) Rs. 22,178

28. If sum to n terms of a series is given by $(n + 8)$, then its second term will be given by

(a) 10 (b) 9
(c) 8 (d) 1
(e) None of these

29. If A is the sum of the n terms of the series 1 + 1/4 + 1/16 + ... and B is the sum of $2n$ terms of the series 1 + 1/2 + 1/4 + ..., then find the value of A/B.

(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4
(e) 4/5

30. A man receives a pension starting with Rs. 100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) Rs. 1100 (b) Rs. 1000
(c) Rs. 1200 (d) Rs. 900
(e) Rs. 1250

31. The sum of the series represented as:
$1/1\times5 + 1/5\times9 + 1/9\times13 \ldots + 1/221\times225$
is

(a) 28/221 (b) 56/221
(c) 56/225 (d) None of these

32. The sum of the series
$1/(\sqrt{2}+\sqrt{1}) + 1/(\sqrt{2}+\sqrt{3}) + \ldots\ldots + 1/(\sqrt{120}+\sqrt{121})$
is:

(a) 10 (b) 11
(c) 12 (d) None of these

33. Find the infinite sum of the series 1/1 +1/3 + 1/6 +1/10 + 1/15 ....

(a) 2 (b) 2.25
(c) 3 (d) 4

34. The sum of the series $5\times8 + 8\times11 + 11\times14$ upto n terms will be:

(a) $(n+1)[3(n+1)^2 + 6(n+1) + 1] - 10$
(b) $(n+1)[3(n+1)^2 + 6(n+1) + 1] + 10$
(c) $(n+1)[3(n+1) + 6(n+1)^2 + 1] - 10$
(d) $(n+1)[3(n+1) + 6(n+1)^2 + 1] + 10$

35. The sum of the series: ½ + 1/6 + 1/12 + 1/20 +...1/156 + 1/182 is:

(a) 12/13 (b) 13/14
(c) 14/13 (d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms:

(a) 1.1 (b) 1
(c) 14/13 (d) None of these

Directions for Questions 37–39: Answer the questions based on the following information.

There are 250 integers $a_1, a_2 \ldots\ldots a_{250}$, not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers a_1 through a_{124} form sequence A, and the rest form sequence B. Each member of A is less than or equal to each member of B.

37. All values in A are changed in sign, while those in B remain unchanged. Which of the following statements is true?

(a) Every member of A is greater than or equal to every member of B.
(b) Max is in A.
(c) If all numbers originally in A and B had the same sign, then after the change of sign, the largest number of A and B is in A.
(d) None of these

38. Elements of A are in ascending order, and those of B are in descending order. a_{124} and a_{125} are interchanged. Then which of the following statements is true?

(a) A continues to be in ascending order.
(b) B continues to be in descending order.
(c) A continues to be in ascending order and B in descending order.
(d) None of the above

39. Every element of A is made greater than or equal to every element of B by adding to each element of A an integer x. Then, x cannot be less than:

32. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours.
   (a) $2^9 - 1$ (b) 2^{10}
   (c) 2^9 (d) $2^{10} - 1$
   (e) $2^{11} - 1$
33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.
   (a) 49637 (b) 39767
   (c) 49634 (d) 39770
   (e) 38770
34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3)/abc$ equals
   (a) $(ab)^{1/2}/c$ (b) 1
   (c) a^2c/b (d) $2a$
   (e) None of these
35. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is
   (a) Positive (b) Negative
   (c) Non-positive (d) Non-negative
   (e) Either (c) or (d)

Hints and Solutions II

2. $a + (a + d) + (a + 2d) + (a + 3d) = 20$
   and $a(a + 3d) = (a + d)(a + 2d)$
4. Calculate the sum of an AP with first term 1, common difference 1 and last term 12. Multiply this sum by 4 for 2 days.
5. The maximum sum will occur when the last term is either 2 or 0.
6. Visualise the AP as 7, 14…196.
9. The AP is 105, 112 …994.
10. The common difference is $\frac{146}{5} = 29.2$.
11. See the terms of the series in 33 blocks of 3 each. This will give the AP – 4, –5, –6…–33. Further, the hundredth term will be 34.
12. Solve through options.
14. The first drop is 120 metres. After this the ball will rise by 96 metres and fall by 96 meters. This process will continue in the form of an infinite GP with common ratio 0.8 and first term 96.
   The required answer will be got by
   $$120 + 96 * 1.25 * 2$$
15. Take any GP and solve by using values.
18. Solve by using values to check options.
22. The difference between the seventh and third term is given by
   $$(a + 6d) - (a + d)$$
23. $\frac{(39-4)}{5} = 7$.
27. The required answer will be by adding 20 terms of the GP starting with the first term as 1000 and the common ratio as 1.05.
30. Visualise it as an infinite GP with common ratio 0.9.

Hints and Solutions III

1. Difference between the tenth and the sixth term = –16
   or $(a + 9d) - (a + 5d) = -16$
   $\rightarrow d = -4$
2. Sum of the first term and the fifth term = 10
   or $a + a + 4d = 10$
   or $a + 2d = 5$ (1)
   and, the sum of all terms of the AP except for the 1st term = 99
   or $9a + 45d = 99$
   $a + 5d = 11$ (2)
   Solve (1) and (2) to get the answer.
3. The second statement gives the equation as $a + 8d = 2(a + 3d) + 6$
   or $a - 2d = 6$
   Now, use the options to find the value of d, and put these values to check the equation obtained from the first statement.
   i.e. $(a + 2d)(a + 5d) = 406$
4. To plant the 1st sapling, Mithilesh will cover 20 m; to plant the 2nd sapling he will cover 40 m and so on. But for the last sapling, he will cover only the

even column wise each column is arranged in an *AP* we can conclude the following:

1[st] row – average 51 – total = 23×51

2[nd] row – average 52 – total 23×52.....

23[rd] row – average 73 – total 23×73

The overall total can be got by using averages as: $23 \times 23 \times 62 = 32798$

42. Total burgers made = 660

Burgers with chicken and mushroom patty = 165 (Number of terms in the series 1, 5, 9...657)

Burgers with vegetable patty = 95 (Number of terms in the series 2, 9, 16, ...660)

Burgers with chicken, mushroom and vegetable patty = 24 (Number of terms in the series 9, 37, 65....653)

Required answer = 660 – 165 – 95 + 24 = 424

43. From the above question, we have 24 such burgers.

LOD 3

9. For 1 term, the value should be:

$$6^2 + 8 = 44$$

Only option (b) gives 44 for $n = 1$

16. The solution (from the options) has got something to do with either 2^{100} or 2^{101} for 100 terms. Hence, for 3 terms recreate the options and crosscheck with the actual sum.

For 3 terms: Sum = 2 + 8 + 24 = 34.

(a) $100 \times 2^{101} + 2$ for 100 terms becomes $3 \times 2^4 + 2$ for 3 terms.

$= 48 + 2 = 50 \neq 34$. Hence is not correct.

(b) $99 \times 2^{100} + 2$ for 100 terms becomes $2 \times 2^3 + 2$ for 3 terms.

But this does not give 34. Hence is not correct.

(c) $99 \times 2^{101} + 2 \rightarrow 2 \times 2^4 + 2 = 34$

(d) $100 \times 2^{100} + 2 \rightarrow 3 \times 2^3 + 2 \neq 34$.

Hence, option (c) is correct.

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. Lata has the same number of sisters as she has brothers, but her brother Shyam has twice as many sisters as he has brothers. How many children are there in the family?
(a) 7 (b) 6 (c) 5 (d) 3
2. How many times does the digit 6 appear when you count from 11 to 100?
(a) 9 (b) 10 (c) 19 (d) 20
3. If $m < n$, then
(a) $m.m < n.n$
(b) $m.m > n.n$
(c) $m.n.n < n.m.m$
(d) $m.m.m < n.n.n$
4. A square is drawn by joining the midpoints of the side of a given square. A third square is drawn in side the second square in the same way and this process is continued indefinitely. If a side of the first square is 8 cm, the sum of the areas of all the squares (in sq. cm) is
(a) 128 (b) 120
(c) 96 (d) None of these
5. Find the least number which when divided by 6, 15, 17 leaves a remainder 1, but when divided by 7 leaves no remainder.
(a) 211 (b) 511 (c) 1022 (d) 86
6. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is
(a) 26 (b) 18 (c) 31
(d) None of these
7. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is
(a) 44 (b) 62 (c) 80 (d) 86
8. An intelligence agency decides on a code of 2 digits selected from 0, 1, 2,....9. But the slip on which the code is hand-written allows confusion between top and bottom, because these are indistinguishable. Thus, for example, the code 91 could be confused with 16. How many codes are there such that there is no possibility of any confusion?
(a) 25 (b) 75
(c) 80 (d) None of these
9. Suppose one wishes to find distinct positive integers x, y such that $(x + y)/xy$ is also a positive integer. Identify the correct alternative.
(a) This is never possible
(b) This is possible and the pair (x, y) satisfying the stated condition is unique.
(c) This is possible and there exist more than one but a finite number of ways of choosing the pair(x, y).
(d) This is possible and the pair (x, y) can be chosen in infinite ways.
10. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her.
(a) thumb (b) index finger
(c) middle finger (d) ring finger
11. 139 persons have signed up for an elimination tournament. All players are to be paired up for the first round, but because 139 is an odd number one player gets a bye, which promotes him to the second round, without actually playing in the first round. The pairing continues on the next round, with a bye to any player left over. If the schedule is planned so that a minimum number of matches is required to determine the champion, the number of matches which must be played is
(a) 136 (b) 137 (c) 138 (d) 139
12. The product of all integers from 1 to 100 will have the following numbers of zeros at the end.
(a) 20 (b) 24 (c) 19 (d) 22
13. There are ten 50 paise coins placed on a table. Six of these show tails four show heads. A coin chosen at random and flipped over (<u>not</u> tossed). This operation is performed seven times. One of the coins is then covered. Of the remaining nine coins five show tails and four show heads. The covered coin shows
(a) a head (b) a tail
(c) more likely a head (d) more likely a tail
14. A five digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?
(a) 6666600 (b) 6666660
(c) 6666666 (d) None

the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$ then the value of b is:

(a) 1 (b) 0 (c) 5 (d) 6
(e) None of these

16. The remainder 7^{84} is divided by 342 is:
(a) 0 (b) 1 (c) 49 (d) 341
(e) None of these

17. Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements can't be true?
(a) $(x - z)^2y$ is even (b) $(x - z)y^2$ is odd
(c) $(x - y)y$ is odd (d) $(x - y)^2z$ is even
(e) None of these

18. A boy starts adding consecutive natural numbers starting with 1. After some time he reaches a total of 1000 when he realizes that he has made the mistake of double counting 1 number. Find the number double counted.
(a) 44 (b) 45
(c) 10 (d) 12
(e) None of these

19. In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to decimal number system, becomes:
(a) 406 (b) 1086
(c) 213 (d) 691
(e) None of these

20. Ashish is given Rs 158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and Rs 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?
(a) 11 (b) 12
(c) 13 (d) 14
(e) None of these

REVIEW TEST FOUR

1. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit.
(a) 648 (b) 528
(c) 728 (d) 128
(e) None of these

2. Which is the highest 3-digit number that divides the number 11111....1(27 times) perfectly without leaving any remainder?
(a) 111 (b) 333 (c) 666 (d) 999
(e) None of these

3. W1, W2, ... , W7 are 7 positive integral values such that by attaching the coefficients of +1, 0 and -1 to each value available and adding the resultant values, any number from 1 to 1093 (both included) could be formed. If W1, W2, ... , W7 are in ascending order, then what is the value of W3?
(a) 10 (b) 9 (c) 0 (d) 1
(e) None of these

4. What is the unit digit of the number $63^{25} + 25^{63}$?
(a) 3 (b) 5 (c) 8 (d) 2
(e) None of these

5. Find the remainder when $(2222^{5555} + 5555^{2222})$ is divided by 7.
(a) 1 (b) 0 (c) 2 (d) 5
(e) None of these

6. What is the number of nines used in numbering a 453 page book?
(a) 86 (b) 87 (c) 84 (d) 83
(e) 85

7. How many four digit numbers are divisible by 5 but not by 25?
(a) 2000 (b) 8000 (c) 1440 (d) 9999
(e) None of these

8. The sum of two integers is 10 and the sum of their reciprocals is 5/12. What is the value of larger of these integers?
(a) 7 (b) 5 (c) 6 (d) 4
(e) None of these

9. Saurabh was born in 1989. His elder brother Siddhartha was also born in the 1980's such that the last two digits of his date of birth form a prime number P. Find the remainder when $(P^2 + 11)$ is divided by 5.
(a) 0 (b) 1 (c) 2 (d) 3
(e) Can't be determined

10. The HCF of x and y is H. Find the HCF of $(x - y)$ and $(x^3 + y^3)/(x^2 - xy + y^2)$.
(a) $H - 1$ (b) H^2 (c) H (d) $H + 1$
(e) None of these

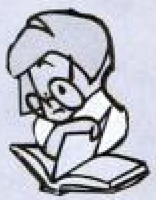
Review Test Two

1.	a
2.	a
3.	b
4.	c
5.	c
6.	a
7.	d
8.	c
9.	b
10.	c
11.	c
12.	d
13.	b
14.	d
15.	a
16.	d
17.	b
18.	d
19.	b
20.	c
21.	c
22.	c
23.	c
24.	d
25.	a

BLOCK 2

CHAPTERS

- **Averages**
- **Alligations**



...BACK TO SCHOOL

- **The Relevance of Averages**

Average is one of the most important mathematical concepts that we use in our day to day life. In fact, even the most non mathematical individuals regularly utilize the concept of averages on a day to day basis.

So, we use averages in all the following and many more instances.

- How a class of students fared in an exam is assessed by looking at the average score.
- What is the average price of items purchased by an individual.
- A person might be interested in knowing his average telephone expenditure, electricity expenditure, petrol expenditure, etc.
- A manager might be interested in finding out the average sales per territory or even the average growth rate month to month.
- Clearly there can be immense application of averages that you might be able to visualize on your own.

- **The Meaning of an Average**

The average is best seen as a representative value which can be used to represent the value of the general term in a group of values.

For instance, suppose that a cricket team had 10 partnerships as follows:

1st wicket 28	2nd wicket 42
3rd wicket 112	4th wicket 52
5th wicket 0	6th wicket 23
7th wicket 41	8th wicket 18
9th wicket 9	10th wicket 15

On adding the ten values above, we get a total of 340—which gives an average of 34 runs per wicket, i.e. the average partnership of the team was 34 runs.

Contd.

the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the questions you have solved for both the chapters.

If You Scored:> 12 (In Unlimited Time)

Follow the same process as above. The only difference is that the school book work is optional – do it only if you feel you need to. However, your concentration during the solving of the two chapters has to be on developing your speed at solving questions on this chapter.

Solution From the first sentence, we get that the total from Monday to Wednesday was 81 while from Tuesday to Thursday was 72. The difference is arising out of the replacement of Monday by Thursday.

This can be mathematically written as

$$\text{Mon} + \text{Tue} + \text{Wed} = 81 \quad (1)$$

$$\text{Tue} + \text{Wed} + \text{Thu} = 72 \quad (2)$$

Hence, $$\text{Mon} - \text{Thu} = 9$$

We have two unknown variables in the above equation. To solve for 2 unknowns, we need a new equation. Looking back at the problem we get the equation:

$$\text{Thu} = (2/3) \times \text{Mon}$$

Solving the two equations we get: Thursday = 18 °C.

However, in the exam, you should avoid using equation-solving as much as possible. You should, ideally, be able to reach half way through the solution during the first reading of the question, and then meet the gap through the use of options.

The answer to this problem should be got by the time you finish reading the question for the first time.

Thus suppose we have the equations:
$M - T = 9$ and $T = 2M/3$ or $T/M = 2/3$ and have the options for T as

(a) 12 (b) 15 (c) 18 (d) 27

To check which of these options is the appropriate value, we need to check one by one.

Option (a) gives $T = 12$, then we have $M = 21$. But $12/21 \neq 2/3$. Hence, this is not the correct option.

Option (b) gives $T = 15$, then $M = 24$. But again $15/24 \neq 2/3$. Hence, this is not the correct option.

Option (c) gives $T = 18$, then $M = 27$. Now $18/27 = 2/3$. Hence, this is the correct option.

So we no longer need to check for option (d).

However, if we had checked for option (d) then $T = 27$, so $M = 36$. But again $27/36 \neq 2/3$. Hence, this is not the correct option.

In the above, we used 'solving-while-reading' and 'option-based' approaches.

These two approaches are very important and by combining the two, you can reach amazing speeds in solving the question.

You are advised to practice both these approaches while solving questions, which will surely improve your efficiency and speed. You will see that, with practice, you will be able to arrive at the solution to most of the LOD I problems (given later in this chapter) even as you finish reading the questions. And since it is the LOD I level problems that appear in most examinations (like CET Maharashtra, Bank PO, MAT, Indo MAT, NMIMS, NIFT, NLS and most other aptitude exams) you will gain a significant advantage in solving these problems.

On LOD II, LOD III and CAT type problems, you will find that using solving-while-reading and option-based approaches together would take you through anywhere between 30–70% of the question by the time you finish reading the question for the first time.

This will give you a tremendous time advantage over the other students appearing in the examination.

Problem 2.6 A person covers half his journey by train at 60 kmph, the remainder half by bus at 30 kmph and the rest by cycle at 10 kmph. Find his average speed during the entire journey.

Solution Recognise that the journey by bus and that by cycle are of equal distance. Hence, we can use the short cut illustrated earlier to solve this part of the problem.

Using the process explained above, we get average speed of the second half of the journey as

$$10 + 1 \times 5 = 15 \text{ kmph}$$

Then we employ the same technique for the first part and get

$$15 + 1 \times 9 = 24 \text{ kmph (Answer)}$$

Problem 2.7 A school has only 3 classes that contain 10, 20 and 30 students respectively. The pass percentage of these classes are 20% , 30% and 40% respectively. Find the pass percentage of the entire school.

Solution

Using weighted average: $\dfrac{10 \times 0.2 + 20 \times 0.3 + 30 \times 0.4}{10 + 20 + 30}$

$= \dfrac{20}{60} = 33.33\%$

Alternatively, we can also use solving-while-reading as

Recognize that the pass percentage would be given by

$$\frac{\text{Passed students}}{\text{Total students}}$$

(a) 64 years (b) 48 years
(c) 45 years (d) 40 years
(e) None of these

45. The average salary of 20 workers in an office is Rs.1900 per month. If the manager's salary is added, the average salary becomes Rs. 2000 per month. What is the manager's annual salary?
(a) Rs. 24,000 (b) Rs. 25,200
(c) Rs. 45,600 (d) RS. 46,000
(e) None of these

46. If a, b, c, d and e are five consecutive odd numbers, then their average is
(a) $5(a + b)$ (b) $(a\ b\ c\ d\ e)/5$
(c) $5(a + b + c + d + e)$ (d) $\frac{a+b}{5}$
(e) None of these

47. The average of first five multiples of 3 is
(a) 3 (b) 9 (c) 12 (d) 15
(e) None of these

48. The average weight of a class of 40 students is 40 kg. If the weight of the teacher be included, the average weight increases by 500 gm. The weight of the teacher is
(a) 40.5 kg (b) 60 kg
(c) 62 kg (d) 60.5 kg
(e) 64 kg

49. In a management entrance test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 120 marks. The number of questions he answered correctly was
(a) 50 (b) 45 (c) 60 (d) 68
(e) None of these

50. The average age of four children is 8 years, which is increased by 4 years when the age of the father is included. Find the age of the father.
(a) 32 (b) 28 (c) 16 (d) 24
(e) 30

51. The average of the first ten natural numbers is
(a) 5 (b) 5.5 (c) 6.5 (d) 6

52. The average of the first ten whole numbers is
(a) 4.5 (b) 5 (c) 5.5 (d) 4

53. The average of the first ten even numbers is
(a) 18 (b) 22 (c) 9 (d) 11

54. The average of the first ten odd numbers is
(a) 11 (b) 10 (c) 17 (d) 9

55. The average of the first ten prime numbers is
(a) 15.5 (b) 12.5 (c) 10 (d) 12.9

56. The average of the first ten composite numbers is
(a) 12.9 (b) 11 (c) 11.2 (d) 10

57. The average of the first ten prime numbers, which are odd, is
(a) 12.9 (b) 13.8 (c) 17 (d) 15.8

58. The average weight of a class of 30 students is 40 kg. If, however, the weight of the teacher is included, the average become 41 kg. The weight of the teacher is
(a) 31 kg (b) 62 kg (c) 71 kg (d) 70 kg

59. Ram bought 2 toys for Rs. 5.50 each, 3 toys for Rs. 3.66 each and 6 toys for Rs. 1.833 each. The average price per toy is
(a) Rs. 3 (b) Rs. 10 (c) Rs. 5 (d) Rs. 9

60. 30 oranges and 75 apples were purchased for Rs. 510. If the price per apple was Rs. 2, then the average price of oranges was
(a) Rs. 12 (b) Rs. 14 (c) Rs. 10 (d) Rs. 15

61. The average income of Sambhu and Ganesh is Rs. 3,000 and that of Arun and Vinay is Rs. 500. What is the average income of Sambhu, Ganesh, Arun and Vinay?
(a) Rs. 1750 (b) Rs. 1850
(c) Rs. 1000 (d) Rs. 2500

62. A batsman made an average of 40 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after fifth inning?
(a) 32 (b) 22 (c) 38 (d) 49

63. The average weight of a school of 40 teachers is 80 kg. If, however, the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?
(a) 109 kg (b) 29 kg
(c) 39 kg (d) None of these

64. The average temperature of 1st, 2nd and 3rd December was 24.4 °C. The average temperature of the first two days was 24 °C. The temperature on the 3rd of December was
(a) 20 °C (b) 25 °C
(c) 25.2 °C (d) None of these

65. The average age of Ram and Shyam is 20 years. Their average age 5 years hence will be

(a) 100 (b) 90 (c) 110 (d) 105
(e) 120

25. The average salary of the entire staff in an office is Rs. 3200 per month. The average salary of officers is Rs. 6800 and that of non-officers is Rs. 2000. If the number of officers is 5, then find the number of non-officers in the office?
(a) 8 (b) 12 (c) 15 (d) 5
(e) 10

26. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. What will be the average speed during the whole journey?
(a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/xyz$
(c) $3xyz/(xy + yz + xz)$ (d) None of these

Directions for Questions 27–30: Read the following passage and answer the questions that follow.

In a family of five persons *A*, *B*, *C*, *D* and *E*, each and everyone loves one another very much. Their birthdays are in different months and on different dates. *A* remembers that his birthday is between 25th and 30th, of *B* it is between 20th and 25th, of *C* it is between 10th and 20th, of *D* it is between 5th and 10th and of *E* it is between 1st to 5th of the month. The sum of the date of birth is defined as the addition of the date and the month, for example 12th January will be written as 12/1 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

27. What may be the maximum average of their sum of the dates of birth?
(a) 24.6 (b) 15.2
(c) 28 (d) 32

28. What may be the minimum average of their sum of the dates of births?
(a) 24.6 (b) 15.2
(c) 28 (d) 32

29. If it is known that the dates of birth of three of them are even numbers then find maximum average of their sum of the dates of birth.
(a) 24.6 (b) 15.2
(c) 27.6 (d) 28

30. If the date of birth of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.
(a) 27.2 (b) 26.4
(c) 28 (d) None of these

31. The average age of a group of persons going for a picnic is 16.75 years. 20 new persons with an average age of 13.25 years join the group on the spot due to which the average of the group becomes 15 years. Find the number of persons initially going for the picnic?
(a) 24 (b) 20 (c) 15 (d) 18

32. A school has only four classes that contain 10, 20, 30 and 40 students respectively. The pass percentage of these classes are 20%, 30%, 60% and 100% respectively. Find the pass % of the entire school.
(a) 56% (b) 76% (c) 34% (d) 66%

33. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ at $x = 10$. $f(x)$ is equal to $x^2 + 2$, $g(x) = 5x^2 - 3$, $h(x) = \log x^2$ and $d(x) = (4/5)x^2$
(a) 170 (b) 170.25 (c) 70.25 (d) 70

34. Find the average of $f(x) - g(x)$, $g(x) - h(x)$, $h(x) - d(x)$, $d(x) - f(x)$
(a) 0 (b) –2.25 (c) 4.5 (d) 2.25

35. $\sum_{r=1}^{n} (n+1)r$ where $r = n$.
(a) $\frac{(n-1)(n)(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$
(c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n^2}{2}$

36. The average of 'n' numbers is z. If the number x is replaced by the number x^1, then the average becomes z^1. Find the relation between n, z, z^1, x and x^1.
(a) $\left[\frac{z^1 - 2}{x^1 - x} = \frac{1}{n}\right]$ (b) $\left[\frac{x^1 - x}{z^1} = \frac{1}{n}\right]$
(c) $\left[\frac{z - z^1}{x - x^1} = \frac{1}{n}\right]$ (d) $\left[\frac{x - x^1}{z - z^1} = \frac{1}{n}\right]$

37. The average salary of workers in AMS careers is Rs. 2,000, the average salary of faculty being Rs. 4,000 and the management trainees being Rs. 1,250. The total number of workers could be
(a) 450 (b) 300 (c) 110 (d) 500

Directions for Questions 38–41: Read the following and answer the questions that follows.

During a criket match, India playing against NZ scored in the following manner:

Partnership	*Runs scored*
1st wicket	112

(c) 10 rows and 50 seats
(d) 50 rows and 10 seats
(e) 40 rows and 20 seats

33. One fashion house has to make 810 dresses and another one 900 dresses during the same period of time. In the first house, the order was ready 3 days ahead of time and in the second house, 6 days ahead of time. How many dresses did each fashion house make a day if the second house made 21 dresses more a day than the first?
(a) 54 and 75 (b) 24 and 48
(c) 44 and 68 (d) 04 and 25
(e) None of these

34. A shop sold 64 kettles of two different capacities. The smaller kettle cost a rupee less than the larger one. The shop made 100 rupees from the sale of large kettles and 36 rupees from the sale of small ones. How many kettles of either capacity did the shop sell and what was the price of each kettle?
(a) 20 kettles for 2.5 rupees each and 14 kettles for 1.5 rupees each
(b) 40 kettles for 4.5 rupees each and 24 kettles for 2.5 rupees each
(c) 40 kettles for 2.5 rupees each and 24 kettles for 1.5 rupees each
(d) either a or b
(e) None of these

35. An enterprise got a bonus and decided to share it in equal parts between the exemplary workers. It turned out, however, that there were 3 more exemplary workers than it had been assumed. In that case, each of them would have got 4 rupees less. The administration had found the possibility to increase the total sum of the bonus by 90 rupees and as a result each exemplary worker got 25 rupees. How many people got the bonus?
(a) 9 (b) 18 (c) 8 (d) 16
(e) 20

Directions for Questions 36–39: Read the following and answer the questions that follows.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

36. If the minister for economic affairs decided to reverse the process of calculation of average income and average expenditure, what will happen to the estimated savings of a person living on Hoola Boola Moola island?
(a) It will increase
(b) It will decrease
(c) It will remain constant
(d) Will depend on the value

37. If it is known that Mr. Magoo Hoola Boola estimates his savings at 10 Moolahs and if it is further known that his actual expenditure is 288 Moolahs in an year (Moolahs, for those who are not aware, is the official currency of Hoola Boola Moola), then what will happen to his estimated savings if he suddenly calculates on the basis of a 12 month calendar year?
(a) Will increase by 5
(b) Will increase by 15
(c) Will increase by 10
(d) Will triple

38. Mr. Boogie Woogie comes back from the USA to Hoola Boola Moola and convinces his community comprising 546 families to start calculating the average income and average expenditure on the basis of 12 months per calendar year. Now if it is known that the average estimated income on the island is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).
(a) 251.60 Moolahs (b) 565.5 Moolahs
(c) 625.5 Moolahs (d) Cannot be determined

39. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income on the basis of 12 months per calendar year. Now if it is known that the average estimated income in his community is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).

7. The maximum possible average for *B* will occur if all the 5 transferees from *A* have 22 marks.
8. The average of Group *A* after the transfer in Q. 7 above is:
(400 – 18*5)/15 = 310/15 = 20.66
9. (400 – 22*5)/15 = 19.33
10. 400 + 23*5 = 515. Average = 515/25 = 20.6
11. 400 + 31*5 = 555. Average = 555/25 = 22.2
12. Will always decrease since the net value transferred from *B* to *A* will be higher than the net value transferred from *A* to *B*.
13. Since the lowest score in Class *B* is 23 which is more than the highest score of any student in Class *A*. Hence, *A*'s average will always increase.
14. The maximum possible value for *B* will happen when the *A* to *B* transfer has the maximum possible value and the reverse transfer has the minimum possible value.
15. For the minimum possible value of *B* we will need the *A* to *B* transfer to be the lowest possible value while the *B* to *A* transfer must have the highest possible value. Thus, *A* to *B* transfer → 18* 5 while *B* to *A* transfer will be 31*5. Hence answer is 22.4.
16. The maximum value for *A* will happen in the case of Q. 15. Then the increment for group *A* is:
31*5 – 18*5 = 5*(31 – 18) = 65.
Thus maximum possible value is 465/20 = 23.25.
17. Minimum possible average will happen for the transfer we saw in Q. 14. Thus the answer will be 405/20 = 20.25.
18. The maximum possible value for *C* will be achieved when the transfer from *C* is of five 26's and the transfer back from *B* is of five 31's. Hence, difference is totals will be +25. Hence, max. average = (900 + 25)/30 = 30.833.
[Note here that 900 has come by 30*30]
19. For the maximum possible value of Class *B* the following set of operations will have to hold:
Five 33's are transferred from *C* to *B*, whatever goes from *B* to *A* comes back from *A* to *B*, then five 23's are transferred from *B* to *C*. This leaves us with:
Increase of 50 marks → average increases by 2 to 27.
20. *A* will attain maximum value if five 33's come to *A* from *C* through *B* and five 18's leave *A*. In such a case the net result is going to be a change of +75. Thus the average will go up by 75/20 = 3.75 to 23.75.

21–23. Will be solved by the same pattern as the above questions.
24. Only option *A* will give us the required situation since the transfer of five 31's increases the value of the average of group *A*.
25–28. Will be solved by the same pattern as above questions.
29–35. These are standard questions using the concept of averages. Hence, analyse each and every sentence by itself and link the interpretations. If you are getting stuck, the only reason is that you have not used the information in the questions fully.
36. Monthly estimates of income is reduced as the denominator is increased from 12 to 14 at the same time the monthly estimate of expenditure is increased as the denominator is reduced from 12 to 9. Hence, the savings will be underestimated.
37–39. Use the averages formulae and common sense to answer.
40–49. The questions are commonsensical with a lot of calculations and assumptions involved. You have to solve these using all the information provided.
40. Das's score = 15*2 + 7*2.5 = 47.5 → 48.
Dasgupta's score = 20*1 + 2*1.5 = 23
41. From the above the answer is 48:23 = 96:46.
42–44. By maximum possible support from the other end, you have to assume that he has Laxman or Sehwag batting aggressively for the entire tenure at the crease. Strike has to be shared equally.
42. Through options, After 60 overs, score would be 150. Then Tendulkar can score @ 4 runs per over (sharing the strike and batting aggressively) and get maximum support @ 3 runs per over. Thus in 30 overs left the target will be achieved.
43. Tendulkar's score for the innings will be 30*4 = 120.
44. We do not know when Laxman would have come into bat. Hence this cannot be determined.
45–49. Build in each of the conditions in the problem to form a table like:

Partnership	Partner	Overs faced	Tendulkar's score	Partner's score
6th wicket	Ganguly	12	6 overs × 6	6 overs × 4
7th wicket	and so on			
8th wicket				
9th wicket				
10th wicket				

4

ALLIGATIONS

INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time wherever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

The relevance of this chapter for the CAT is the use of the alligation technique for solving problems where the concept is used to solve a part of the whole question. Besides, questions of this chapter are directly relevant for the exams like CET (Maharashtra), NMIMS, NIFT, Symbiosis, MAT and all other B level management entrance exams as well as for the bank PO exams and the MCA exams, which are based on the pattern of an aptitude test.

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1 for group	1	containing	n_1	elements
A_2 for group	2	containing	n_2	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that the weighted average, Aw is given by:

$$Aw = (n_1A_1 + n_2 A_2 + n_3 A_3 + \ldots\ldots. + n_kA_k)/ (n_1 + n_2 + n_3 \ldots + n_k)$$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$$Aw = (n_1A_1 + n_2 A_2)/(n_1 + n_2)$$

Rewriting this equation we get: $(n_1 + n_2)\ Aw = n_1A_1 + n_2A_2$

$$n_1(Aw - A_1) = n_2\ (A_2 - Aw)$$

or $n_1/n_2 = (A_2 - Aw)/(Aw - A_1) \rightarrow$ The alligation equation.

The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2, then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

goods sold. Find his percentage profit on the whole. (26%)

5. A car travels at 20 km/h for 40 minutes and at 30 km/h for 60 minutes. Find the average speed of the car for the journey. (26 km/hr)
6. 40% of the revenues of a school came from the junior classes while 60% of the revenues of the school came from the senior classes. If the school raises its fees by 20% for the junior classes and by 30% for the senior classes, find the percentage increase in the revenues of the school. (26%)

Some Keys to spot A_1, A_2 and Aw and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This isn't foolproof though, since at times the question might confuse the student by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and Aw are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n_1 and n_2).
4. All percentage values represent the average values.

A Typical Problem

A typical problem related to the topic of alligation goes as follows:

4 litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated 3 times

(a) What is the final quantity of wine left in the cask.
(b) What is the ratio of wine to water finally.

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus at the end of the second step we get: 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 3.24 litres wine and 0.76 litres water leaving 29.16 litres of wine and 10.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can get the amount of wine left by:

$$40 \times 36/40 \times 36/40 \times 36/40 = 40 \times (36/40)^3$$
$$\Rightarrow 40 \times (1 - 4/40)^3$$

This yields the formula:

Wine left : Capacity × (1 – fraction of wine withdrawn)^n for n operations.

Thus, you could have multiplied:

$40 \times (0.9)^3$ to get the answer

That is, reduce 40 by 10% successively thrice to get the required answer.

Thus, the thought process could be:

40 – 10% → 36 – 10% → 32.4 – 10% → 29.16

Level of Difficulty (LOD) I

(Please note that there are no LOD II and LOD III in this chapter since the questions are based on only certain basic situations that cannot be extended.)

1. If 5 kg of salt costing Rs. 5/kg and 3 kg of salt costing Rs. 4/kg are mixed, find the average cost of the mixture per kilogram.
   (a) Rs. 4.5 (b) Rs. 4.625
   (c) Rs. 4.75 (d) Rs. 4.125
   (e) Rs. 4.25
2. Two types of oils having the rates of Rs. 4/kg and Rs. 5/kg respectively are mixed in order to produce a mixture having the rate of Rs. 4.60/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?
   (a) 75 kg (b) 50 kg
   (c) 60 kg (d) 40 kg
   (e) None of these
3. How many kilograms of sugar worth Rs. 3.60 per kg should be mixed with 8 kg of sugar worth Rs. 4.20 per kg, such that by selling the mixture at Rs. 4.40 per kg, there may be a gain of 10%?
   (a) 6 kg (b) 3 kg (c) 2 kg (d) 5 kg
   (e) 4 kg

ANSWER KEY

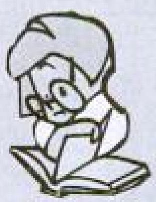
Alligations LOD I

1. b	33. d	
2. c	34. d	
3. e	35. b	
4. e	36. b	
5. d	37. c	
6. a	38. c	
7. e	39. c	
8. b	40. a	
9. c		
10. c		
11. b		
12. c		
13. a		
14. d		
15. c		
16. c		
17. a		
18. c		
19. d		
20. c		
21. a		
22. b		
23. d		
24. a		
25. c		
26. b		
27. c		
28. d		
29. c		
30. c		
31. c		
32. b		

BLOCK 3

CHAPTERS

- Percentages
- Profit and Loss
- Interest
- Ratio, Proportion and Variation
- Time and Work
- Time, Speed and Distance



...BACK TO SCHOOL

As you are already aware, this block consists of the following chapters:

Percentages,
Profit & Loss,
Ratio & Proportion,
Interests,
Time and Work,
Time, Speed and Distance

To put it very simply, the reason for these seemingly diverse chapters to be under one block of chapters are:

Linear Equations

Yes, the solving of linear equations is the common thread that binds all the chapters in this block.

But before we start going through what a linear equation is, let us first understand the concept of a variable and it's need in the context of solving mathematical expressions.

Let us start off with a small exercise first:

Think of a number.

Contd.

PREASSESSMENT TEST

1. Three runners A, Band C run a race, with runner A finishing 24 metres ahead of runner B and 36 metres ahead of runner C, while runner B finishes 16 metres ahead of runner C. Each runner travels the entire distance at a constant speed. .What was the length of the race?
   (a) 72 metres (b) 96 metres
   (c) 120 metres (d) 144 metres
2. A dealer buys dry fruits at Rs.100, Rs. 80 and Rs. 60 per kilogram. He mixes them in the ratio 4:5:6 by weight, and sells at a profit of 50%. At what price per kilogram does he sell the dry fruit?
   (a) Rs.116 (b) Rs.106
   (c) Rs.115 (d) None of these
3. There are two containers: the first contains 500 ml of alcohol, while the second contains 500 ml of water. Five cups of alcohol from the first container is taken out and is mixed well in the second container. Then, five cups of this mixture is taken out from the second container and put back into the first container. Let X and Y denote the proportion of alcohol in the first and the proportion of water in the second container. Then what is the relationship between X & Y? (Assume the size of the cups to be identical)
   (a) X>Y (b) X<Y
   (c) X=Y (d) Cannot be determined
4. Akhilesh took five papers in an examination, where each paper was of 200 marks. His marks in these' papers were in the proportion of 7: 8: 9 :10 : 11. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is:
   (a) 1 (b) 3
   (c) 4 (d) 5
5. A and B walk up an escalator (moving stairway). The escalator moves at a constant speed, A takes six steps for every four of B's steps. A gets to the top of the escalator after having taken 50 steps, while B (because his slower pace lets the escalator do a little more of the work) takes only 40 steps to reach the top. If the escalator were turned off, how many steps would they have to take to walk up?
   (a) 80 (b) 100
   (c) 120 (d) 160
6. Fifty per cent of the employees of a certain company are men, and 80% of the men earn more than Rs. 2.5 lacs per year. If 60% of the company's employees earn more than Rs. 2.5 lacs per year, then what fraction of the women employed by the company earn more than Rs. 2.5 lacs per year?
   (a) 2/5 (b) 1/4
   (c) 1/3 (d) 3/4
7. A piece of string is 80 centimeters long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized and the shortest piece is 46 centimeters shorter than the longest piece. Find the length of the shortest piece (in cm).
   (a) 14 (b) 10
   (c) 8 (d) 18
8. Three members of a family A, B, and C, work together to get all household chores done. The time it takes them to do the work together is six hours less than A would have taken working alone, one hour less than B would have taken alone, and half the time C would have taken working alone. How long did it take them to do these chores working together?
   (a) 20 minutes (b) 30 minutes
   (c) 40 minutes (d) 50 minutes
9. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes?
   (a) 2kg (b) 2.4kg
   (c) 2.5kg (d) None of these
10. At the end of the year 2008, a shepherd bought twelve dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p> 0 and q >0. If the shepherd had twelve dozen goats at the end of the year 2012, (after making the sales for that year), which of the following is true?
    (a) p = q (b) p<q
    (c) p>q (d) p = q/2

Directions for Questions 11–12: Answer the questions based on the following information.

An Indian company purchases components X and Y from UK and Germany, respectively. X and Y form 40% and 30% of the total production cost. Current gain is 25%. Due to change in the international exchange rate scenario, the cost of the German mark increased by 50% and that of

In case the growth in your score is not significant, go back to the theory of both the chapters and re solve all LODs of all the chapters. While doing so concentrate more on the LOD 2 & LOD 3 questions.

If You Scored 15+ (In Unlimited Time)

Obviously you are much better than the first two category of students. Hence unlike them, your focus should be on developing your speed by picking up the shorter processes explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you:

Step One: Quickly review the concepts given in the block three Back to School Section. Only go deeper into a concept in case you find it new. Going back to school level books is not required for you.

Step Two: Move into each of the chapters of the block three one by one.

When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 & LOD 2 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 & 2. While doing so try to work on faster processes for solving the same questions. Concentrate on how you could have solved the same question faster. Also try to think of how much time you took over the calculations.

Step Three: Go to the first review test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

Step Four: Go to the second review test given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

Step Five: In case the growth in your score is not significant (esp. under time limits), review each of the LOD 1 & LOD 2 questions for all the chapters.

Step Six: Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 & LOD 2. Repeat the process that you followed in LOD 1 – going into each chapter one by one.

Step Seven: Go to the remaining review tests given at the end of the block and solve them. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

3. Product constancy application
4. A → B→A application
5. Denominator change to Ratio Change application
6. Use of PCG to calculate Ratio Changes

Application 1: PCG Applied to Successive Changes

This is a very common situation in most questions.
Suppose you have to solve a question in which a number 30 has two successive percentage increases (20% and 10% respectively).

The situation is handled in the following way using PCG:

$$30 \xrightarrow[+6]{20\% \text{ increase}} 36 \xrightarrow[+3.6]{10\% \text{ increase}} 39.6$$

Illustration

A's salary increases by 20% and then decreases by 20%. What is the net percentage change in *A*'s salary?

Solution:

$$100 \xrightarrow[+20]{20\% \text{ inc.}} 120 \xrightarrow[-24]{20\% \text{ decrease}} 96$$

Hence, *A*'s salary has gone down by 4%

Illustration

A trader gives successive discounts of 10%, 20% and 10% respectively. The percentage of the original cost price he will recover is:

Solution:

$$100 \xrightarrow[-10]{10\% \text{ decrease}} 90 \xrightarrow[-18]{20\% \text{ decrease}} 72 \xrightarrow[-7.2]{10\% \text{ decrease}} 64.8$$

Hence the overall discount is 35.2% and the answer is 64.8%.

Illustration

A trader marks up the price of his goods by 20%, but to a particularly haggling customer he ends up giving a discount of 10% on the marked price. What is the percentage profit he makes?

Solution:

$$100 \xrightarrow[+20]{20\% \text{ increase}} 120 \xrightarrow[-12]{10\% \text{ decrease}} 108$$

Hence, the percentage profit is 8%.

Application 2: PCG applied to Product Change

Suppose you have a product of two variables say 10 × 10.

If the first variable changes to 11 and the second variable changes to 12, what will be the percentage change in the product? [Note there is a 10% increase in one part of the product and a 20% increase in the other part.]

The formula given for this situation goes as: $(a + b + ab/100)$

Hence, Required % change = $10 + 20 + \frac{10 \times 20}{100}$

(Where 10 and 20 are the respective percentage changes in the two parts of the product) (This is being taught as a shortcut at most institutes across the country currently.)

However, a much easier solution for this case can be visualized as:

$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+12]{10\% \uparrow} 132$. Hence, the final product shows a 32% increase.

Similarly suppose 10 × 10 × 10 becomes 11 × 12 × 13
In such a case the following PCG will be used:

$$100 \xrightarrow[+30]{30\% \uparrow} 130 \xrightarrow[+26]{20\% \uparrow} 156 \xrightarrow[+15.6]{10\% \uparrow} 171.6$$

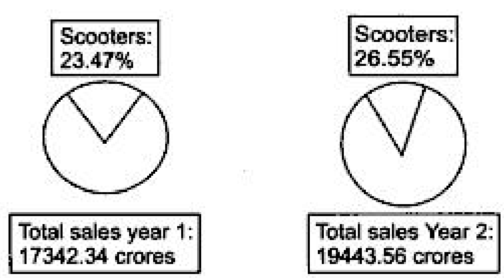
Hence, the final product sees a 71.6 percent increase (Since, the product changes from 100 to 171.6)

Note: You will get the same result irrespective of the order in which you use the respective percentage changes.

Also note that this process is very similar to the one used for calculating successive percentage change.

Application for DI:

Suppose you have two pie charts as follows:



If you are asked to calculate the percentage change in the sales revenue of scooters for the company from year one to year two, what would you do?

The formula for percentage change would give us:

$$\frac{(0.2655 \times 19443.56) - (0.2347 \times 17342.34) \times 100}{(0.2347 \times 17342.34)}$$

shifting the decimal point by two places to the left. Thus, 83.33% = 0.8333 in decimal value.

- A second learning from this table is in the process of division by any of the numbers such as 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 15, 16, 24 and so on, students normally face problems in calculating the decimal values of these divisions. However, if one gets used to the decimal values that appear in the Table 5.2, calculation of decimals in divisions will become very simple. For instance, when an integer is divided by 7, the decimal values can only be .14, .28, .42, .57, .71, .85 or .00. (There are approximate values)
- This also means that the difference between two ratios like $\frac{x}{6} - \frac{x}{7}$ can be integral if and only if x is divisible by both 6 and 7.

  This principle is very useful as an advanced short cut for option based solution of some questions. I leave it to the student to discover applications of this principle.

Calculation of Multiplication by Numbers like 1.21, 0.83 and so on

In my opinion, the calculation of multiplication of any number by a number of the form 0.*xy* or of the form 1.*ab* should be viewed as a subtraction/addition situation and not as a multiplication situation. This can be explained as follows.

Example: Calculate 1.23 × 473.

Solution: If we try to calculate this by multiplying, we will end up going through a very time taking process, which will yield the final value at the end but nothing before that (i.e. you will have no clue about the answer's range till you reach the end of the calculation).

Instead, one should view this multiplication as an addition of 23% to the original number. This means, the answer can be got by adding 23% of the number to itself.

Thus 473 × 1.23 = 473 + 23% of 473 = 473 + 94.6 + 3% of 473 = 567.6 + 14.19 = 581.79

(The percentage rule can be used to calculate the addition and get the answer.)

The similar process can be utilised for the calculation of multiplication by a number such as 0.87

(Answer can be got by subtracting 13% of the number from itself and this calculation can again be done by percentage rule.)

Hence, the student is advised to become thorough with the percentage rules. Percentage calculation & additions of 2 & 3 digit numbers.

WORKED-OUT PROBLEMS

Problem 5.1 A sells his goods 30% cheaper than *B* and 30% dearer than *C*. By what percentage is the cost of *C*'s goods cheaper than *B*'s goods.

Solution There are two alternative processes for solving this question:

1. Assume the price of *C*'s goods as *p*.: Then *A*'s goods are at 1.3 p and *B*'s goods are such that *A*'s goods are 30% cheaper than *B*'s goods. i.e. *A*'s goods are priced at 70% of *B*'s goods.

Hence, 1.3 $p \to 70$

B's price $\to 100$

B's price = 130 p/70 = 1.8571 p

Then, the percentage by which *C*'s price is cheaper than *B*'s price =

$(1.8571\ p - p) \times 100/(1.8571\ p) = 600/13 =$ **46.15%**

Learning task for student Could you answer the question: Why did we assume *C*'s price as a variable *p* and then work out the problem on its basis. What would happen if we assumed *B*'s price as *p* or if we assumed *A*'s price as *p*?

2. Instead of assuming the price of one of the three as p, assume the price as 100.

Let *B* = 100. Then *A* = 70, which is 30% more than *C*. Hence *C* = 23.07% less than *A* (from Table 4.1) = approx. 53.84. Hence answer is 46.15% approximately.

(This calculation can be done mentally if you are able to work through the calculations by the use of percentage rule. The students are advised to try to assume the value of 100 for each of the variables *A*, *B* and *C* and see what happens to the calculations involved in the problem. Since the value of 100 is assumed for a variable to minimise the requirements of calculations to solve the problems, we should ensure that the variable assumed as 100 should have the maximum calculations associated with it.)

(c) 21.23% (d) 27.27%
(e) None of these

20. The price of sugar is reduced by 25% but inspite of the decrease, Aayush ends up increasing his expenditure on sugar by 20%. What is the percentage change in his monthly consumption of sugar ?
(a) +60% (b) –10% (c) +33.33% (d) 50%
(e) None of these

21. The price of rice falls by 20%. How much rice can be bought now with the money that was sufficient to buy 20 kg of rice previously?
(a) 5 kg (b) 15 kg (c) 25 kg (d) 30 kg
(e) 20 kg

22. 30% of a number when subtracted from 91, gives the number itself. Find the number.
(a) 60 (b) 65
(c) 70 (d) 75
(e) None of these

23. When 60% of a number *A* is added to another number *B*, *B* becomes 175% of its previous value. Then which of the following is true regarding the values of *A* and *B*?
(a) $A > B$
(b) $B > A$
(c) $B \geq A$
(d) Either (a) or (b) can be true depending upon the values of *A* and *B*
(e) Nothing can be said

24. At an election, the candidate who got 56% of the votes cast won by 144 votes. Find the total number of voters on the voting list if 80% people cast their vote and there were no invalid votes.
(a) 360 (b) 720 (c) 1800 (d) 1500
(e) 1600

25. The population of a village is 1,00,000. The rate of increase is 10% per annum. Find the population at the start of the third year?
(a) 1,33,100 (b) 1,21,000
(c) 1,18,800 (d) 1,20,000
(e) None of these

26. The population of the village of Gavas is 10,000 at this moment. It increases by 10% in the first year. However, in the second year, due to immigration, the population drops by 5%. Find the population at the end of the third year if in the third year the population increases by 20%.
(a) 12,340 (b) 12,540
(c) 1,27,540 (d) 12,340
(e) 13,240

27. A man invests Rs. 10,000 in some shares in the ratio 2 : 3 : 5 which pay dividends of 10%, 25% and 20% (on his investment) for that year respectively. Find his dividend income.
(a) 1900 (b) 2000
(c) 2050 (d) 1950
(e) 1850

28. In an examination, Mohit obtained 20% more than Sushant but 10% less than Rajesh. If the marks obtained by Sushant is 1080, find the percentage marks obtained by Rajesh if the full marks is 2000.
(a) 86.66% (b) 72%
(c) 78.33% (d) 77.77%
(e) None of these

29. In a class, 25% of the students were absent for an exam. 30% failed by 20 marks and 10% just passed because of grace marks of 5. Find the average score of the class if the remaining students scored an average of 60 marks and the pass marks are 33 (counting the final scores of the candidates).
(a) 37.266 (b) 37.6
(c) 37.8 (d) 36.93
(e) 37.5

30. Ram spends 20% of his monthly income on his household expenditure, 15% of the rest on books, 30% of the rest on clothes and saves the rest. On counting, he comes to know that he has finally saved Rs. 9520. Find his monthly income.
(a) 10000 (b) 15000
(c) 20000 (d) 12000
(e) None of these

31. Hans and Bhaskar have salaries that jointly amount to Rs. 10,000 per month. They spend the same amount monthly and then it is found that the ratio of their savings is 6 : 1. Which of the following can be Hans's salary?
(a) Rs. 6000 (b) Rs. 5000
(c) Rs. 4000 (d) Rs. 3000

32. The population of a village is 5500. If the number of males increases by 11% and the number of females increases by 20%, then the population becomes 6330. Find the population of females in the town.
(a) 2500 (b) 3000
(c) 2000 (d) 3500

rural areas. If the FDI in Gujarat, which goes to urban areas, is \$72 m, then find the size of FDI in rural Andhra Pradesh, which attracts 50% of the FDI that comes to Andhra Pradesh, which accounts for 20% of the total FDI?

(a) \$30 m (b) \$9 m
(c) \$60 m (d) \$40 m
(e) None of these

17. If in question 16, the growth in the size of FDI for the next year with respect to the previous year is 20%, then find the share of urban Maharashtra next year if 12% of the total FDI going to Maharashtra went to urban areas (provided Maharashtra attracted only 10% of the total share for both years).

(a) \$36 m (b) \$4.32 m
(c) \$3 m (d) \$5 m
(e) None of these

18. The cost of food accounted for 25% of the income of a particular family. If the income gets raised by 20%, then what should be the percentage point decrease in the food expenditure as a percentage of the total income to keep the food expenditure unchanged between the two years?

(a) 3.5 (b) 8.33 (c) 4.16 (d) 5
(e) 6.25

19. If the length, breadth and height of a cube are decreased, decreased and increased by 5%, 5% and 20% respectively, then what will be the impact on the surface area of the cube (in percentage terms)?

(a) 7.25% (b) 5% (c) 8.33% (d) 6.0833%
(e) 8.5%

20. *A*'s salary is first increased by 25% and then decreased by 20%.The result is the same as *B*'s salary increased by 20% and then reduced by 25%. Find the ratio of *B*'s salary to that of *A*'s?

(a) 4 : 3 (b) 11 : 10
(c) 10 : 9 (d) 12 : 11
(e) None of these

21. The minimum quantity of milk in litres (in whole number) that should be mixed in a mixture of 60 litres in which the initial ratio of milk to water is 1 : 4 so that the resulting mixture has 15% milk is

(a) 3 (b) 4
(c) 5 (d) 6
(e) This is not possible

22. A person saves 6% of his income. Two years later, his income shoots up by 15% but his savings remain the same. Find the hike in his expenditure.

(a) 15.95% (b) 15%
(c) 14.8% (d) 15.5%
(e) None of these

23. *A* is 50% more than *B*, *C* is 2/3 of *A* and *D* is 60% more than *C*. Now, each of *A*, *B*, *C* and *D* is increased by 10%. Find what per cent of *B* is *D* (after the increase)?

(a) 150% (b) 160%
(c) 175% (d) 176%
(e) None of these

24. *A* and *B* have, between them, Rs. 1200. *A* spends 12% of his money while *B* spends 20% of his money. They are then left with a sum that constitutes 85% of the whole sum. Find what amount is left with *A*.

(a) Rs. 750 (b) Rs. 800
(c) Rs. 700 (d) Rs. 660
(e) Rs. 880

25. Maya has Rs. *M* with her and her friend Chanda has Rs. *C* with her. Maya spends 12% of her money and Chanda also spends the same amount as Maya did. What percentage of her money did Chanda spend?

(a) $\frac{18M}{C}$ (b) $\frac{18C}{M}$
(c) $\frac{12M}{C}$ (d) $\frac{12C}{M}$
(e) None of these

26. In a village consisting of *p* persons, *x*% can read and write. Of the males alone *y*%, and of the females alone *z*% can read and write. Find the number of males in the village in terms of *p*, *x*, *y* and *z* if $z < y$.

(a) $\frac{p(x-z)}{(y+x-z)}$ (b) $\frac{p(x-z)}{(y+x-2z)}$
(c) $\frac{p(y-x)}{(x-z)}$ (d) $\frac{p(x-z)}{(y-z)}$

27. In order to maximise his gain, a theatre owner decides to reduce the price of tickets by 20% and as a result of this, the sales of tickets increase by 40%. If, as a result of these changes, he is able to increase his weekly collection by Rs. 1,68,000, find by what value did the gross collection increase per day?

(a) 14,000 (b) 18,000
(c) 24,000 (d) 20,000

10. In question 8, if Vawal wants to limit the increase in expenditure to Rs. 200, what strategy should he adopt with respect to his travel?
    (a) Reduce travel to 2350 kilometres
    (b) Reduce travel to 2340 kilometres
    (c) Reduce travel to 2360 kilometres
    (d) Reduce travel to 2370 kilometres
    (e) None of these
11. A shopkeeper announces a discount scheme as follows: for every purchase of Rs. 3000 to Rs. 6000, the customer gets a 15% discount or a ticket that entitles him to get a 7% discount on a further purchase of goods costing more than Rs. 6000. The customer, however, would have the option of reselling his right to the shopkeeper at 4% of his initial purchase value (as per the right refers to the 7% discount ticket). In an enthusiastic response to the scheme, 10 people purchase goods worth Rs. 4000 each. Find the maximum. Possible revenue for the shopkeeper.
    (a) Rs. 38,400 (b) Rs. 38,000
    (c) Rs. 39,400 (d) Rs. 39,000
    (e) Rs. 40,000
12. For question 11, find the maximum possible discount that the shopkeeper would have to offer to the customer.
    (a) Rs. 1600 (b) Rs. 2000
    (c) Rs. 6000 (d) Rs. 4000
    (e) None of these

Directions for Questions 13–16: Read the following and answer the questions that follow.

Two friends Shayam and Kailash own two versions of a car. Shayam owns the diesel version of the car, while Kailash owns the petrol version.

Kailash's car gives an average that is 20% higher than Shayam's (in terms of litres per kilometre). It is known that petrol costs 60% of its price higher than diesel.

13. The ratio of the cost per kilometre of Kailash's car to Shayam's car is
    (a) 3 : 1 (b) 1 : 3
    (c) 1.92 : 1 (d) 2 : 1
    (e) Cannot be determined
14. If Shayam's car gives an average of 20 km per litre, then the difference in the cost of travel per kilometre between the two cars is
    (a) Rs. 4.3 (b) Rs. 3.5
    (c) Rs. 2.5 (d) Rs. 3
    (e) Cannot be determined
15. For question 14, the ratio of the cost per kilometre of Shayam's travel to Kailash's travel is
    (a) 3 : 1 (b) 1 : 3
    (c) 1 : 1.92 (d) 2 : 1
    (e) Cannot be determined
16. If diesel costs Rs. 12.5 per litre, then the difference in the cost of travel per kilometre between Kailash's and Shayam's is (assume an average of 20 km per litre for Shayam's car and also assume that petrol is 50% of its own price higher than diesel)
    (a) Rs. 1.75 (b) Rs. 0.875
    (c) Rs. 1.25 (d) Rs. 1.125
    (e) None of these

Directions for Questions 17–23: Read the following and answer the questions that follow.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

17. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income and the average expenditure on the basis of 12 months per calendars year. Now if it is known that the average estimated income in his community is (according to the old system) 87 moolahs per month, then what will be the percentage change in the savings of the community of Mr. Boogle Woogle (assume that there is no other change)?
    (a) 12.33% (b) 22.22%
    (c) 31.31% (d) 33.33%
    (e) Cannot be determined
18. For question 17, if it is known that the average estimated monthly expenditure is 19 moolahs per month for the island of Hoola Boola Moola, then what will be the percentage change in the estimated savings of the community?
    (a) 32.42% (b) 38.05%
    (c) 25.23% (d) 26.66%
    (e) Cannot be determined

(c) 14.02% (d) None of these

46. If the change in production is directly related to the change in investment in the previous year, and if the data of the savings rate change for the previous 2 questions are to be assumed true, then for which year did the difference between the production in the Indian economy and the production in the Pakistani economy show the maximum percentage change?

(a) 2006–07 (b) 2007–08
(c) 2008–09 (d) Cannot be determined

47. For question 44, it is known that the percentage change in investment in a year leads to a corresponding equal percentage increase in the manufacturing production in the next year. Further, if the growth rate of manufacturing production is 27% of the GDP growth rate of the country, then what is the GDP growth rate of India in 2006–07?

(a) 8.52% (b) 7.28%
(c) 9.26% (d) None of these

48. The Euro was ushered in on the Ist January 2002 and the old currencies of the European economies were exchanged into Euros. In France, 4 Francs were exchanged for 1 Euro while in Germany 5 Deutsche Marks were exchanged for 1 Euro and in Italy 3 Liras were exchanged for 1 Euro. The exchange rate for Moolahs, the official currency of Hoola Boola Moola, was set at 185 Moolahs per Euro. Dr. Krishna Iyer, an NRI doctor based in Europe, had a practice across each of these three countries and he sends back money orders to his native island of Hoola Boola Moola. The existing exchange rate of Moolahs with the above-mentioned currencies was 51 moolahs per Franc, 36 Moolahs per Deutsche Mark and 70 moolahs per Lira. If Dr. Iyer has this information, then what should he do with his currency holdings in these three currencies on the 31st December 2001 so that he maximises his moolah value on the Ist of January 2002. (Assume no arbitrage possibilities between the three currencies)

(a) Change to Francs
(b) Change to Deutsche Marks
(c) Change to Liras
(d) Remain indifferent

49. For the above questions, the exchange rates for the three currencies with respect to a dollar was: 2$ per Lira, 1.5$ per Franc and 1.4 dollar per Deutshce Mark. If Dr. Iyer has 100 liras, 100 Deutsche Marks and 100 Francs on 31st December 2001, the maximum percentage change he can achieve in his net holding in terms of dollars due to the arbitrage created by the Euro conversion could be

(a) 17.23% (b) 7.33%
(c) 11.2% (d) Cannot be determined

50. For question 48, which one of the following will allow the calcutation of all possibilities of percentage change in terms of moolah value of Dr. Iyer's portfolio. That is possible through currency conversions.

(a) Dr. Iyer's money holding in all three currencies
(b) Dr. Iyer's monthly earnings in all three currencies
(c) The inter-currency conversion rates between Liras, Deutsche Mark and Francs
(d) Both a and c

Hints and Solutions II

1. Assume the initial value to be 100 and solve.
3. Total number of students = Full fee waver + 50% concession + No concession.
6. Assume initial value of price = 100.
   Since, the price is a mutiplicative function, we have
   $100 \times 1.1 \times 1.2 \times 0.8 \times 1.25 \times 1.5$
   Solve using percentage change graphic.
8. Salary ratio is 2.25 : 2.2.6666.
   Hence 0.41666 = Rs. 40.
   Then, 6.9166 = Rs. 664
9. Solve using options
13. Assume initial amount of gold to be 100.
    Then he gives away: 50 + 25 + 12.5 = 87.5
    But 87.5 = 1309000 kg
    Hence, 100 = 149600
18. If initial income = 100, initial food expenditure = 25.
    New income = 120
    Since, food expenditure is constant at 25, the percentage of the new income = 20.833.
    Percentage point change = 25 – 20.833 = 4.166
24. Solve using options.
25. Use standard formulae of percentage.

Hence, I am being offered a discount of Rs. 6 on Rs. 42 – a 14.28% discount. Hence, the second shirt is a better bargain.

45. 72% must have voted for Sonia Gandhi and 16% for Sushma Swaraj. Hence, 88 × 3 = 264.

48. $100 \xrightarrow[\text{Sales increase}]{50\% \uparrow} 150 \xrightarrow[\text{Price drop}]{?} 82.5$ (final sales figure)

Hence, the required price drop is 67.5/150 = 45% drop

50. In 2001, BMW = 15%, Maruti = 50% and hence Honda = 35%

LOD II

1. $100 \longrightarrow 150 \longrightarrow \underset{(\text{year } 1)}{75} \longrightarrow 112.5 \longrightarrow \underset{(\text{year } 2)}{56.25}$

$\longrightarrow 84.375 \longrightarrow 42.1875$

Now, 42.1875 = Rs. 16,875

Hence $1 \longrightarrow 400$

Also, year 2 donation is 56.25 × 400 = 22500

3. The thought process would go like:

If we assume 100 students

Total : 60 boys and 40 girls.

Fee waiver : 9 boys and 3 girls.

This means that a total of 12 people are getting a fee waiver. (But this figure is given as 90.)

Hence, 1 corresponds to 7.5.

Now, number of students not getting a fee waiver = 51 boys and 37 girls

50% concession → 25.5 boys and 18.5 girls (i.e. a total of 44.)

Hence, the required answer = 44 × 7.5 = 330

4. Solve using options. Checking for option (b), gives us:

$$200000 \rightarrow 180000 \rightarrow 171000 \rightarrow 153900 \rightarrow 146205$$

(by consecutively decreasing 200000 by 10% and 5% alternately)

5. Total characters in her report = 25 × 60 × 75.

Let the new no. of pages be n.

Then:

$$n \times 55 \times 90 = 25 \times 60 \times 75$$
$$n = 22.72$$

This means that her report would require 23 pages. A drop of 8% in terms of the pages.

7. The total raise of salary is 87.5% (That is what 15/8 means here).

Using the options and PCG, you get option (c) as the correct answer.

8. October : November : December = 9 : 8 : 10.666 since, he got Rs 40 more in December than October, we can conclude that 1.666 = 40 → 1 = 24.

Thus, total Bonus for the three months is:

$$0.4 \times 27.666 \times 24 = 265.6$$

10. 9% increase is offset by 8.26% decrease. Hence, option (b) is correct.

11. The expenditure increase can be calculated using PCG as:

$$100 \rightarrow 112.5 \rightarrow 123.75.$$

A 23.75% increase.

14. Population at the start = 100.

Population after 2 years = 100 × 1.08 × 1.01 × 1.08 × 1.01 = 108.984

Thus, the required percentage increase = 18.984%

16. 24% of the total goes to urban Gujarat $72 m

∴ 1% = $ 3 mn.

The required value for Rural AP

= 50% of 20% = 10%

Hence, required answer = $ 30 mn

17. In the previous question, the total FDI was $ 300 mn. A growth of 20% this year means a total FDI of $360 mn.

The required answer is 12% of 10% of 360 mn

= 1.2% of 360 = $4.32 mn.

18. The income goes to 120. Food expenditure has to be maintained at 25. (i.e. 20.833%)

Hence, percentage point drop from 25 to 20.833 is 4.16%

25. Chanda would have spent 12% of Maya

Thus, her percentage of expenditure would be 0.12 M × 100/C = 12 M/C

27. The weekly change is equal to Rs. 1,68,000.

Hence, the daily collection will go up by 1,68,000/7 = 24,000.

29. Solve using options. 2/25 fits the requirement.

32. If Z = 100, X = 80 and Y = 72.

Thus, Y is less then X by 10%

34. The correct answer should satisfy the following condition: If 'x' is the increased salary

$$x \times 0.8 \times 0.1 = (x - 4800) \times 0.8 \times 0.12.$$

None of the first 3 options satisfies this.

Thus, it is evident that by converting DM into Liras the increase in value is higher than that achieved by converting DM into Francs.

Similarly, converting Francs to Liras also increases the value of the Francs.

1200 × 51 becomes equivalent to 900 × 70.

Note: The thought process goes like this: 1200 Francs = 300 Euros (since 1 euro = 4 francs). Further 300 Euros equals 900 liras which equal 900 × 70 Moolas.

49. Cannot be determined since the conversion from dollar to Euro is not given, neither is the inter currency exchange rate between Lira, Francs and DMs.
50. Obviously, both a and c are required in order to answer this question.

6

PROFIT AND LOSS

INTRODUCTION

Traditionally, Profit & Loss has always been an important chapter for CAT. Besides, all other Management entrance exams like SNAP, CET, MAT, ATMA as well as Bank P.O. exams extensively use questions from this chapter. From the point of view of CAT, the relevance of this chapter has been gradually reducing and infact there have been no questions from this chapter for the past 3 years. However, CAT being a highly unpredictable exam, my advice to students and readers would be to go through this chapter and solve it at least up to LOD 2, so that they are ready for any changes in patterns.

Further, the Level of Difficulties at which questions are set in the various exams can be set as under:

LOD I: CAT, XLRI, IRMA, IIFT, CET, Bank PO aspirants, MAT, NIFT, NMIMS, and FMS, Symbiosis and all other management exams.

LOD II: CAT, XLRI, FMS, IRMA (partially), etc.

LOD III: CAT, XLRI and FMS (students aiming for 60% plus in Maths in CAT).

THEORY

Profit and Loss are part and parcel of every commercial transaction. In fact, the entire economy and the concept of capitalism is based on the so called "Profit Motive".

Profit and Loss in Case of Individual Transactions

We will first investigate the concept of Profit and Loss in the case of individual transactions. Certain concepts are important in such transaction. They are:

The price at which a person buys a product is the cost price of the product for that person. In other words, the amount paid or expended in either purchasing or producing an object is known as its Cost Price (also written as CP).

The price at which a person sells a product is the sales price of the product for that person. In other words, the amount got when an object is sold is called as the *Selling Price* (*SP*) of the object from the seller's point of view.

When a person is able to sell a product at a price higher than its cost price, we say that he has earned a profit. That is,

If SP > CP, the difference, SP – CP is known as the profit or gain.

Similarly, if a person sells an item for a price lower than its cost price, we say that a loss has been incurred.

The basic concept of profit and loss is as simple as this.

If, however, SP < CP, then the difference, CP – SP is called the loss.

It must be noted here that the Selling Price of the seller is the Cost Price of the buyer.

Thus we can say that in the case of profit the following formulae hold true:

1. Profit = SP – CP
2. SP = Profit + CP
3. CP = SP – Profit
4. Percentage Profit = $\frac{\text{Profit} \times 100}{\text{CP}}$

Percentage Profit is always calculated on CP unless otherwise stated.

assumed CP does not make the SP equal 2000 it means that the assumed CP is incorrect. Hence, you should move to the next option. Use logic to understand whether you go for the higher options or the lower options based on your rejection of the assumed option.

Note: The above question will never appear as a full question in the examination but might appear as a part of a more complex question. If you are able to interpret this statement through the denominator change to ratio change table, the time requirement will reduce significantly and you will gain a significant time advantage over this statement.

Problem 6.2 A man buys a shirt and a trouser for Rs. 371. If the trouser costs 12% more than the shirt, find the cost of the shirt.

Solution Here, we can write the equation:
$s + 1.12s = 371 \rightarrow s = 371/2.12$ (however, this calculation is not very easily done)

An alternate approach will be to go through options. Suppose the options are

(a) Rs. 125 (b) Rs. 150 (c) Rs. 175 (d) Rs. 200

Checking for, say, Rs. 150, the thought process should go like:

Let s = cost of a shirt

If $s = 150$, $1.12s$ will be got by increasing s by 12% i.e. 12% of 150 = 18. Hence the value of $1.12s = 150 + 18 = 168$ and $s + 1.12s = 318$ is not equal to 371. Hence check the next higher option.

If $s = 175$, $1.12s = s + 12\%$ of $s = 175 + 21 = 196$. i.e. $2.12\ s = 371$.

Hence, the option is correct.

Problem 6.3 A shopkeeper sells two items at the same price. If he sells one of them at a profit of 10% and the other at a loss of 10%, find the percentage profit/loss.

Generic question: A shopkeeper sells two items at the same price. If he sells one of them at a profit of x% and the other at a loss of x%, find the percentage profit/loss.

Solution The result will always be a loss of $[x/10]^2$%. Hence, the answer here is $[10/10]^2\% = 1\%$ loss.

Problem 6.4 For the Problem 6.3. find the value of the loss incurred by the shopkeeper if the price of selling each item is Rs. 160.

Solution When there is a loss of 10% $\rightarrow$ 160 = 90% of CP_1. $\therefore CP_1 = 177.77$

When there is a profit of 10% $\rightarrow$ 160 = 110% of CP_2 $\therefore CP_2 = 145.45$

Hence total cost price = 177.77 + 145.45 = 323.23 while the net realisation is Rs. 320.

Hence loss is Rs. 3.23.

Short cut for calculation: Since by selling the two items for Rs. 320 the shopkeeper gets a loss of 1% (from the previous problem), we can say that Rs. 320 is 99% of the value of the cost price of the two items. Hence, the total cost is given by 320/0.99 (solution of this calculation can be approximately done on the percentage change graphic).

Problem 6.5 If by selling 2 items for Rs. 180 each the shopkeeper gains 20% on one and loses 20% on the other, find the value of the loss.

Solution The percentage loss in this case will always be $(20/10)^2 = 4\%$ loss.

We can see this directly as 360 $\rightarrow$ 96% of the CP $\rightarrow$ CP = 360/0.96. Hence, by percentage change graphic 360 has to be increased by 4.166 per cent = 360 + 4.166% of 360 = 360 + 14.4 + 0.6 = Rs. 375.

Hence, the loss is Rs. 15.

Problem 6.6 By selling 15 mangoes, a fruit vendor recovers the cost price of 20 mangoes. Find the profit percentage.

Solution Here since the expenditure and the revenue are equated, we can use percentage profit = goods left × 100/ goods sold = 5 × 100/15 = 33.33%.

Problem 6.7 A dishonest shopkeeper uses a 900 gram weight instead of 1 kilogram weight. Find his profit percent if he sells per kilogram at the same price as he buys a kilogram.

Solution Here again the money spent and the money got are equal. Hence, the percentage profit is got by goods left × 100/goods sold.

This gives us 11.11%.

Problem 6.8 A manufacturer makes a profit of 15% by selling a colour TV for Rs. 6900. If the cost of manufacturing increases by 30% and the price paid by the retailer

5. A shopkeeper sold goods for Rs. 2400 and made a profit of 25% in the process. Find his profit per cent if he had sold his goods for Rs. 2040.
   (a) 6.25% (b) 7%
   (c) 6.20% (d) 6.5%
   (e) 6.75%
6. A digital diary is sold for Rs. 935 at a profit of 10%. What would have been the actual profit or loss on it, if it had been sold for Rs. 810?
   (a) Rs. 45 (b) Rs. 40 (c) Rs. 48 (d) Rs. 50
   (e) Rs. 56
7. A music system when sold for Rs. 4500 gives a loss of 16.66% to the merchant who sells it. Calculate his loss or gain per cent, if he sells it for Rs. 5703.75.
   (a) Loss of 5.625% (b) Profit of 8.33%
   (c) Loss of 7% (d) Profit of 5.625%
   (e) Profit of 7%
8. By selling bouquets for Rs. 63, a florist gains 5%. At what price should he sell the bouquets to gain 10% on the cost price?
   (a) Rs. 66 (b) Rs. 69
   (c) Rs. 72 (d) Rs. 72.50
   (e) Rs. 75
9. A shopkeeper bought 240 chocolates at Rs. 9 per dozen. If he sold all of them at Re. 1 each, what was his profit per cent?
   (a) 66(1/6)% (b) 33(1/3)%
   (c) 24% (d) 27%
   (e) None of these
10. A feeding bottle is sold for Rs. 120. Sales tax accounts for one-fifth of this and profit one-third of the remainder. Find the cost price of the feeding bottle.
    (a) Rs. 64 (b) Rs. 72 (c) Rs. 68 (d) Rs. 76
    (e) Rs. 78
11. A coal merchant makes a profit of 20% by selling coal at Rs. 25 per quintal. If he sells the coal at Rs. 22.50 per quintal, what is his profit per cent on the whole investment?
    (a) 6% (b) 6.66% (c) 7.5% (d) 8%
    (e) 9%
12. The cost price of a shirt and a pair of trousers is Rs. 371. If the shirt costs 12% more than the trousers, find the cost price of the trouser.
    (a) Rs. 125 (b) Rs. 150
    (c) Rs. 175 (d) Rs. 200
    (e) Rs. 225
13. A pet shop owner sells two puppies at the same price. On one he makes a profit of 20% and on the other he suffers a loss of 20%. Find his loss or gain per cent on the whole transaction.
    (a) Gain of 4% (b) No profit no loss
    (c) Loss of 10% (d) Loss of 4%
    (e) None of these
14. The marked price of a table is Rs. 1200, which is 20% above the cost price. It is sold at a discount of 10% on the marked price. Find the profit per cent.
    (a) 10% (b) 8% (c) 7.5% (d) 6%
    (e) 8.33%
15. 125 toffees cost Rs. 75. Find the cost of one million toffees if there is a discount of 40% on the selling price for this quantity.
    (a) Rs. 3,00,000 (b) Rs. 3,20,000
    (c) Rs. 3,60,000 (d) Rs. 4,00,000
    (e) Rs. 4,50,000
16. A shopkeeper marks the price of an article at Rs. 80. Find the cost price if after allowing a discount of 10% he still gains 20% on the cost price.
    (a) Rs. 53.33 (b) Rs. 70
    (c) Rs. 75 (d) Rs. 60
    (e) Rs. 66
17. In the question 16, what will be the selling price of the article if he allows two successive discounts of 5% each.
    (a) Rs. 72 (b) Rs. 72.20
    (c) Rs. 75 (d) Rs. 71.66
    (e) Rs. 71.2
18. A dozen pairs of gloves quoted at Rs. 80 are available at a discount of 10%. Find how many pairs of gloves can be bought for Rs. 24.
    (a) 4 (b) 5 (c) 6 (d) 8
    (e) 7
19. Find a single discount equivalent to the discount series of 20%, 10%, 5%.
    (a) 30% (b) 31.6% (c) 68.4% (d) 35%
    (e) 32.6%
20. The printed price of a calculator is Rs. 180. A retailer pays Rs. 137.7 for it by getting successive discounts of 10% and another rate which is illegible. What is the second discount rate?
    (a) 12% (b) 12.5% (c) 15% (d) 20%
    (e) 16.66%

2. A manufacturer estimates that on inspection 12% of the articles he produces will be rejected. He accepts an order to supply 22,000 articles at Rs. 7.50 each. He estimates the profit on his outlay including the manufacturing of rejected articles, to be 20%. Find the cost of manufacturing each article.
   (a) Rs. 6 (b) Rs. 5.50
   (c) Rs. 5 (d) Rs. 4.50
   (e) Rs. 6.5
3. The cost of setting up the type of a magazine is Rs. 1000. The cost of running the printing machine is Rs. 120 per 100 copies. The cost of paper, ink and so on is 60 paise per copy. The magazines are sold at Rs. 2.75 each. 900 copies are printed, but only 784 copies are sold. What is the sum to be obtained from advertisements to give a profit of 10% on the cost?
   (a) Rs. 730 (b) Rs. 720
   (c) Rs. 726 (d) Rs. 736
   (e) Rs. 750
4. A tradesman fixed his selling price of goods at 30% above the cost price. He sells half the stock at this price, one-quarter of his stock at a discount of 15% on the original selling price and rest at a discount of 30% on the original selling price. Find the gain percent altogether.
   (a) 14.875% (b) 15.375%
   (c) 15.575% (d) 16.375%
   (e) 16.5%
5. A tradesman marks an article at Rs. 205 more than the cost price. He allows a discount of 10% on the marked price. Find the profit percent if the cost price is Rs. x.
   (a) $\dfrac{\left[\dfrac{x}{(18450)}-10\right]}{x}$ (b) $\dfrac{[(18450)]-10x}{x}$
   (c) $\dfrac{\left[\dfrac{x}{(18450)}-100\right]}{x}$ (d) $\dfrac{\left[\dfrac{18450}{x}-100\right]}{x}$
   (e) None of these
6. Dolly goes to a shop to purchase a doll priced at Rs. 400. She is offered 4 discount options by the shopkeeper. Which of these options should she opt for to gain maximum advantage of the discount offered.
   (a) Single discount of 30%
   (b) 2 successive discounts of 15% each
   (c) 2 successive discounts of 20% and 10%
   (d) 2 successive discounts of 20% and 12%
   (e) Either (b) or (c)
7. A dishonest dealer marks up the price of his goods by 20% and gives a discount of 10% to the customer. He also uses a 900 gram weight instead of a 1 kilogram weight. Find his percentage profit due to these maneuvers.
   (a) 8% (b) 12%
   (c) 20% (d) 16%
   (e) None of these
8. A dishonest dealer marks up the price of his goods by 20% and gives a discount of 10% to the customer. Besides, he also cheats both his supplier and his buyer by 100 grams while buying or selling 1 kilogram. Find the percentage profit earned by the shopkeeper
   (a) 20% (b) 25%
   (c) 32% (d) 27.5%
   (e) None of these
9. For question 8, if it is known that the shopkeeper takes a discount of 10% from his supplier and he disregards this discount while marking up (i.e. he marks up at the undiscounted price), find the percentage profit for the shopkeeper if there is no other change from the previous problem.
   (a) 32% (b) 36.66%
   (c) 40.33% (d) 46.66%
   (e) 50%
10. Cheap and Best, a kirana shop bought some apples at 4 per rupee and an equal number at 5 per rupee. He then sold the entire quantity at 9 for 2 rupees. What is his percentage profit or loss?
    (a) 1.23% loss (b) 6.66%
    (c) 8.888% (d) No profit no loss
    (e) None of these
11. A watch dealer sells watches at Rs. 600 per watch. However, he is forced to give two successive discounts of 10% and 5% respectively. However, he recovers the sales tax on the net sale price from the customer at 5% of the net price. What price does a customer have to pay him to buy the watch.
    (a) Rs. 539.75 (b) Rs. 539.65
    (c) Rs. 538.75 (d) Rs. 538.65
    (e) Rs. 540.55
12. Deb bought 100 kg of rice for Rs. 1100 and sold it at a loss of as much money as he received for 20 kg rice. At what price did he sell the rice?
    (a) Rs. 9 per kg (b) Rs. 9.1666 per kg
    (c) Rs. 9.5 per kg (d) Rs. 10.33 per kg
    (e) None of these

him. Find his overall profit or loss if he gets no return on unsold items and it is known that a printer costs 50% of a monitor.

(a) Loss of Rs. 48,500 (b) Loss of 21,000
(c) Loss of Rs. 41,000 (d) Inadequate data

48. For question 47, Manish's approximate percentage profit or loss is

(a) 14.37% loss (b) 16.5% loss
(c) 12.14% loss (d) Insufficient information

49. An orange vendor makes a profit of 20% by selling oranges at a certain price. If he charges Rs. 1.2 higher per orange he would gain 40%. Find the original price at which he sold an orange.

(a) Rs. 5 (b) Rs. 4.8
(c) Rs. 6 (d) None of these

50. The AMS magazine prints 5000 copies for Rs. 5,00,000 every month. In the July issue of the magazine, AMS distributed 500 copies free. Besides, it was able to sell 2/3 of the remaining magazines at 20% discount. Besides, the remaining magazines were sold at the printed price of the magazine (which was Rs. 200). Find the percentage profit of AMS in the magazine venture in the month of July (assume a uniform 20% of the sale price as the vendor's discount and also assume that AMS earns no income from advertising for the issue).

(a) 56% (b) 24.8% (c) 28.5% (d) 22.6%

Level of Difficulty (LOD) III

The charges of a taxi journey are decided on the basis of the distance covered and the amount of the waiting time during a journey. Distance wise, for the first 2 kilometres (or any part thereof) of a journey, the metre reading is fixed at Rs. 10 (if there is no waiting). Also, if a taxi is boarded and it does not move, then the meter reading is again fixed at Rs. 10 for the first ten minutes of waiting. For every additional kilometre the meter reading changes by Rs. 5 (with changes in the metre reading being in multiples of Re. 1 for every 200 metres travelled). For every additional minute of waiting, the meter reading changes by Re. 1. (no account is taken of a fraction of a minute waited for or of a distance less than 200 metres travelled). The net meter reading is a function of the amount of time waited for and the distance travelled.

The cost of running a taxi depends on the fuel efficiency (in terms of mileage/litre), depreciation (straight line over 10 years)and the driver's salary (not taken into account if the taxi is self owned).

Depreciation is Rs. 100 per day everyday of the first 10 years. This depreciation has to be added equally to the cost for every customer while calculating the profit for a particular trip. Similarly, the driver's daily salary is also apportioned equally across the customers of the particular day. Assume, for simplicity, that there are 50 customers every day (unless otherwise mentioned). The cost of fuel is Rs. 15 per litre (unless otherwise stated).

The customer has to pay 20% over the meter reading while settling his bill. Also assume that there is no fuel cost for waiting time (unless otherwise stated).

Based on the above facts, answer the following:

1. If Sardar Preetpal Singh's taxi is 14 years old and has a fuel efficiency of 12 km/litre of fuel, find his profit in a run from Howrah Station to Park Street (a distance of 7 km) if the stoppage time is 8 minutes. (Assume he owns the taxi)

(a) Rs. 32.25 (b) Rs. 40.85
(c) Rs. 34.25 (d) Rs. 42.85
(e) Rs. 44.85

2. For question 2, Sardar Preetpal Singh's percentage profit is

(a) 391.42% (b) 380%
(c) 489.71% (d) 438.23%
(e) 444.25%

3. For the same journey as in question 1 if on another day, with heavier traffic, the waiting time increases to 13 minutes, find the percentage change in the profit.

(a) 12% (b) 14%
(c) 13% (d) 16%
(e) 17%

4. For question 3, if Sardar Preetpal Singh idled his taxi for 7 minutes and if the fuel consumption during idling is 50 ml per minute, find the percentage decrease in the profits.

(a) 10.74% (b) 11.21%
(c) 10.87% (d) 9.94%
(e) 10.44%

Directions for Questions 5–10: Answer questions based on this additional information:

Mr. Vikas Verma owns a fleet of 3 taxis, where he pays his driver Rs. 3000 per month. He also insists on keeping an attendant for Rs. 1500 per month in each of his taxis.

(b) Sell maximum monthlies
(c) Sell maximum dailies
(d) Cannot be determined

35. Without the restriction mentioned in the problem above, what should the newspaper vendor do to maximise his profits if his capital is limited?
(a) Sell maximum weeklies
(b) Sell maximum monthlies
(c) Sell maximum dailies
(d) Cannot be determined

36. A fruit vendor buys fruits from the fruit market at wholesale prices and sells them at his shop at retail prices. He operates his shop 30 days a month, as a rule. He buys in multiples of 100 fruits and sells them in multiples of a dozen fruits. He purchases mangoes for Rs. 425 per hundred and sells at Rs. 65 per dozen, he buys apples at Rs. 150 per hundred and sells at Rs. 30 per dozen, he buys watermelons (always of equal size) at Rs. 1800 per hundred and sells at Rs. 360 per dozen. Which of the three fruits yields him the maximum percentage profit?
(a) Mangoes (b) Apples
(c) Watermelons (d) Both b and c

37. For question 36, if he adds oranges, which he buys at Rs. 180 per hundred and sells at Rs. 33 per dozen, what can be his maximum profit on a particular day if he invests Rs. 1800 in purchasing fruits everyday and he sells everything that he buys?
(a) Rs. 1200 (b) Rs. 1180
(c) Rs. 1260 (d) Rs. 1320

38. For questions 36 and 37, if the fruit vendor hires you as a consultant and pays you 20% of his profit in the month of July 2006 as a service charge, what can be the maximum fees that you will get for your consultancy charges?
(a) Rs. 7200 (b) Rs. 14,400
(c) Rs. 7440 (d) Cannot be determined

39. A newspaper costs Rs. 11 to print on a daily basis. Its sale price (printed) is Rs. 3. The newspaper gives a sales incentive of 40% on the printed price, to the newspaper vendors. The newspaper makes up for the loss through advertisements, which are charged on the basis of per column centimetre rates. The advertisement rates of the newspaper are Rs. 300 per cc (column centimetre). It has to give an incentive of 15% on the advertising bill to the advertising agency. If the newspaper has a circulation of 12,000 copies, what is the approximate minimum advertising booking required if the newspaper has to break-even on a particular day. (Assume there is no wastage.)
(a) 300 cc (b) 350 cc (c) 435 cc (d) 450 cc

40. For question 39, if it is known that the newspaper house is unable to recover 20% of its dues, what would be the approximate advertising booking target on a particular day in order to ensure the break-even point?
(a) 375 cc (b) 438 cc
(c) 544 cc (d) 562.5 cc

Hints and Solutions II

1. Cost of 750 articles = Rs. 450 → outlay.
Hence, SP of 600 articles = Rs. 630. (@40% profit on his outlay)
∴ SP of each article = Rs. 1.05.
2. When the manufacturer makes 100 articles, he sells only 88.
Revenues = 88×7.5 = Rs. 660
But profit being 20% of outlay, total outlay is Rs. 550.
3. Total cost = Type + Running cost of printing machine + paper, ink
$= 1000 + 120 \times 9 + 540 = 2620$
∴ Net sum to be recovered = Rs. 2882.
4. Assume CP of goods = 100.
Then marked price = 130.
Revenues $= \frac{1}{2} \times 130 + \frac{1}{4} \times 0.85 \times 130 + \frac{1}{4} \times 0.7 \times 130$
5. If CP = x, then marked price = $x + 205$ and
Selling price = $0.9x + 184.5$.
Profit = $184.5 - 0.1x$
7. Assume CP as 1 Re/gram.
Then, he sells 900 grams for Rs. 1080,
While the CP of 900 grams is Rs. 900.
8. He buys 1100 grams for Rs.1000 and sells 900 grams for Rs. 1080.
To calculate profit percentage, either equate the money or the goods.
9. He buys 1100 grams for Rs. 900 and sells 900 grams for Rs. 1080.
To calculate percentage profit, either equate the money or equate the goods.

The highest value of revenue is seen at a price of Rs. 198.

14 & 15: Using options from question 15. Suppose she had spent Rs. 6 at the market complex, she would spend Rs. 3 at her uncle's shop. The other condition (that she gets 2 sweets less per rupee at the market complex) gets satisfied in this scenario if she had bought 12 chocolates overall. In such a case, her buying would have been 2 per Rupee at the market and 4 per rupee at Uncle Scrooge's shop.

Trial and error will show that this condition is not satisfied for any other option combination.

16. The given situation fits if we take Q as 60% profit and then the loss would be 37.5% (which is 62.5%) of Q). Thus, if Rs. 24 is the cost price, the selling price should be 24 × 1.6 = Rs. 38.8

18. Ramu's total discount:

    8% on 8000 = Rs. 640
    5% on 12000 = Rs. 600
    3% on 16000 = Rs. 480

    Total = Rs. 1720 on Rs. 36000.

    Hence, Realised value = 34280.

    Shyamu's Discounts:

    7% on 12000 = 840
    6% on 8000 = 480
    5% on 16000 = 800

    Rs. 2120 on Rs. 36000

    Hence, Realised value = 33880.

    The higher profit is for Ramu.

    Also, the CP has a mark up of 25% for the Marked price. Thus the CP must have been 28800 (This is got by 36000 – 20% of 36000 – PCG thinking)

    Thus, the profit % for Ramu would be: (5480*100)/28800 → 19% approx.

19. In the case of the given defaults, the discount for Ramu would have gone down to:

    4% on 12000 (the second payment) and the second discount would thus have been Rs.480 meaning that the sale price would have risen by Rs.120 (since there is a Rs.120 drop in the discount)

    1% on 16000 → A reduction of 2% of 16000 in the discount → a reduction of Rs. 320.

    Hence, Ramu's profit would have gone up by Rs.440 in all & would yield his new profit as:

    5480 + 440 = Rs. 5920

20. The following working would show the answer:

    Ramu's Discounts

    7% on 8000 = Rs. 560
    4% on 12000 = Rs. 480
    2% on 16000 = Rs. 320

    Total = Rs. 1360 on Rs. 36,000.

    Shyamu's discounts:

    6% on 12000 = 720
    5% on 8000 = 400
    4% on 16000 = 640

    Rs. 1760 on Rs. 36000

    Thus, their profits would vary by Rs. 400 (since their cost price is the same)

21. If their CP's had been 1000 each, their respective SP's can be calculated as follows:

    $1000 \xrightarrow[= +250]{25\% \uparrow} 1250$ (for the person calculating profit on the CP)

    $1000 \xrightarrow[= +333.33]{33.33\% \uparrow} 1333.33$

    (for the person calculating his profit on SP: 25% of SP = 33.33% of CP)

    The value of the difference in this case has turned out to be Rs.83.33. This has occurred when we have assumed the CP as 1000. But, we are given a difference of Rs.100.

    Use unitary method as follows:

    Difference of 83.33 when CP is 1000

    Hence, difference of 100 when CP = ???

    $$\frac{100 \times 1000}{83.33} = \text{Rs.}1200$$

23. Find out the total revenue realization for both the cases:

    Case 1: (Old) Total sales revenue = 2000 × 3.25 × 0.75.

    Profit_{old} = Total sales revenue – 4800

    Case 2: (New) Total sales revenue = 3000 × 4.25 × 0.75

    Profit_{new} = Total sales revenue – 4800

    The ratio of profit will be given by $\text{Profit}_{new} / \text{Profit}_{old}$

25. The successive discounts must have been of 10% each. The required price will be got by reducing 25 by 10% twice consecutively. (use PCG application for successive change)

Profit and Loss LOD II

1. c	33. b	
2. b	34. c	
3. c	35. a	
4. b	36. b	
5. b	37. b	
6. a	38. d	
7. c	39. d	
8. c	40. c	
9. d	41. c	
10. a	42. c	
11. d	43. a	
12. b	44. c	
13. a	45. a	
14. a	46. a	
15. a	47. a	
16. d	48. a	
17. c	49. d	
18. a	50. b	
19. a		
20. c		
21. a		
22. b		
23. b		
24. c		
25. d		
26. d		
27. b		
28. a		
29. c		
30. d		
31. c		
32. a		

DEPRECIATION OF VALUE

The value of a machine or any other article subject to wear and tear, decreases with time.

This decrease is called its *depreciation.*

Thus if V_0 is the value at a certain time and $r\%$ per annum is the rate of depreciation per year, then the value V_1 at the end of t years is

$$V_1 = V_0\left[1-\frac{r}{100}\right]^t.$$

POPULATION

The problems on Population change are similar to the problems on Compound Interest. The formulae applicable to the problems on compound interest also apply to those on population. The only difference is that in the application of formulae, the annual rate of change of population replaces the rate of compound interest.

However, unlike in compound interest where the rate is always positive, the population can decrease. In such a case, we have to treat population change as we treated depreciation of value illustrated above.

The students should see the chapter on interests essentially as an extension of the concept of percentages. All the rules of percentage calculation, which were elucidated in the chapter of percentages, will apply to the chapter on interests. Specifically, in the case of compound interests, the percentage rule for calculation of percentage values will be highly beneficial for the student.

Besides, while solving the questions on interests the student should be aware of the possibility of using the given options to arrive at the solution. In fact, I feel that the formulae on Compound Interest (CI) unnecessarily make a very simple topic overly mathematical. Besides, the CI formulae are the most unusable formulae available in this level of mathematics since it is virtually impossible for the student to calculate a number like 1.08 raised to the power 3, 4, 5 or more.

Instead, in my opinion, you should view CI problems simply as an extension of the concept of successive percentage increases and tackle the calculations required through approximations and through the use of the percentage rule of calculations.

Thus, a calculation: 4 years increase at 6% pa CI on Rs. 120 would yield an expression: 120×1.06^4. It would be impossible for an average student to attempt such a question and even if one uses advanced techniques of calculations, one will end up using more time than one has. Instead, if you have to solve this problem, you should look at it from the following percentage change graphic perspective:

$$120 \xrightarrow[=7.2]{+6\%} 127.2 \xrightarrow[6+1.62]{+6\%}$$

$$134.82 \xrightarrow[6+2.1]{+6\%} 142.92 \xrightarrow[6+2.58]{+6\%} 15.15 \text{ (approx.)}$$

If you try to check the answer on a calculator, you will discover that you have a very close approximation. Besides, given the fact that you would be working with options and given sufficiently comfortable options, you need not calculate so closely; instead, save time through the use of approximations.

APPLICATIONS OF INTERESTS IN D.I.

The difference between Simple Annual growth rate and Compound Annual Growth Rate:

The Measurement of Growth Rates is a prime concern in business and Economics. While a manager might be interested in calculating the growth rates in the sales of his product, an economist might be interested in finding out the rate of growth of the GDP of an economy.

In mathematical terms, there are basically two ways in which growth rates are calculated. To familiarize yourself with this, consider the following example.

The sales of a brand of scooters increase from 100 to 120 units in a particular city. What does this mean to you? Simply that there is a percentage increase of 20% in the sales of the scooters. Now read further:

What if the sales moves from 120 to 140 in the next year and 140 to 160 in the third year? Obviously, there is a constant and uniform growth from 100 to 120 to 160 – i.e. a growth of exactly 20 units per year. In terms of the overall growth in the value of the sales over there years, it can be easily seen that the sale has grown by 60 on 100 i.e. 60% growth.

In this case, what does 20% represent? If you look at this situation as a plain problem of interests 20% represents the simple interest that will make 100 grow to 160.

Hence, principal = 704 – 64 = 640

Problems 7.5 A sum of money was invested at SI at a certain rate for 3 years. Had it been invested at a 4% higher rate, it would have fetched Rs. 480 more. Find the principal.

(a) Rs. 4000 (b) Rs. 4400 (c) Rs. 5000 (d) Rs. 3500

Solution Let the rate be $y\%$ and principal be Rs. x and the time be 3 years.

Then according to the question = $(x(y + 4) \times 3)/100 - (xy \times 3)/100 = 480$

$\Rightarrow \quad xy + 4x - xy = 160 \times 100$

$\Rightarrow \quad x = (160 \times 100)/4 =$ Rs. 4000

Alternatively: Excess money obtained = 3 years @ 4% per annum

= 12% of whole money

So, according to the question, 12% = Rs. 480

So, 100% = Rs. 4000 (answer arrived at by using unitary method.)

Problem 7.6 A certain sum of money trebles itself in 8 years. In how many years it will be five times?

(a) 22 years (b) 16 years (c) 20 years (d) 24 years

Solution It trebles itself in 8 years, which makes interest equal to 200% of principal.

So, 200% is added in 8 years.

Hence, 400%, which makes the whole amount equal to five times of the principal, which will be added in 16 years.

Problem 7.7 If CI is charged on a certain sum for 2 years at 10% the amount becomes 605. Find the principal?

(a) Rs. 550 (b) Rs. 450 (c) Rs. 480 (d) Rs. 500

Solution Using the formula, amount = Principal $(1 + \text{rate}/100)^{\text{time}}$

$$605 = p(1 + 10/100)^2 = p(11/10)^2$$
$$p = 605(100/121) = \text{Rs. } 500$$

Alternatively: Checking the options,

Option (a) Rs. 550

First year interest = Rs. 55, which gives the total amount Rs. 605 at the end of first year. So not a valid option.

Option (b) Rs. 450

First year interest = Rs. 45

Second year interest = Rs. 45 + 10% of Rs. 45 = 49.5

So, amount at the end of 2 years = 450 + 94.5 = 544.5

So, not valid.

Hence answer has to lie between 450 and 550 (since 450 yields a shortfall on Rs. 605 while 550 yields an excess.)

Option (c) Rs. 480

First year interest = Rs. 48

Second year interest = Rs. 48 + 10% of Rs. 48 = 52.8

So, amount at the end of 2 years = 580.8 ≠ 605

Option (d) Rs. 500

First year's interest = Rs. 50

Second year's interest = Rs. 50 + 10% of Rs. 50

= Rs. 55.

∴ Amount = 605.

Note: In general, while solving through options, the student should use the principal of starting with the middle (in terms of value), more convenient option. This will often reduce the number of options to be checked by the student, thus reducing the time required for problem solving drastically. In fact, this thumb rule should be used not only for the chapter of interests but for all other chapters in maths.

Furthermore, a look at the past question papers of exams like CET, Maharashtra, Lower level MBA exams and bank PO exams will yield that by solving through options and starting with the middle more convenient option, there will be significant time savings for these exams where the questions are essentially asked from the LOD I level.

Problem 7.8 If the difference between the CI and SI on a certain sum of money is Rs. 72 at 12 per cent per annum for 2 years, then find the amount.

(a) Rs. 6000 (b) Rs. 5000 (c) Rs. 5500 (d) Rs. 6500

Solution Let the principal = x

Simple interest = $(x \times 12 \times 2)/100$

Compound interest = $x[1 + 12/100]^2 - x$

So, $x[112/100]^2 - x - 24x/100 = 72$

$x[112^2/100^2 - 1 - 24/100] = 72 \Rightarrow x[12544/10000 - 1 - 24/100] = 72$

$\Rightarrow x = 72 \times 10000/144 =$ Rs. 5000

Alternatively: Simple interest and compound interest for the first year on any amount is the same.

28. A sum of money is borrowed and paid back in two equal annual instalments of Rs. 882, allowing 5% compound interest. The sum borrowed was
(a) Rs. 1640 (b) Rs. 1680
(c) Rs. 1620 (d) Rs. 1700

29. Two equal sums were borrowed at 8% simple interest per annum for 2 years and 3 years respectively. The difference in the interest was Rs. 56. The sum borrowed were
(a) Rs. 690 (b) Rs. 700
(c) Rs. 740 (d) Rs. 780

30. In what time will the simple interest on Rs. 1750 at 9% per annum be the same as that on Rs. 2500 at 10.5% per annum in 4 years?
(a) 6 years and 8 months
(b) 7 years and 3 months
(c) 6 years
(d) 7 years and 6 months

31. In what time will Rs. 500 give Rs. 50 as interest at the rate of 5% per annum simple interest?
(a) 2 years (b) 5 years
(c) 3 years (d) 4 years

32. Shashikant derives an annual income of Rs. 688.25 from Rs. 10,000 invested partly at 8% p.a. and partly at 5% p.a. simple interest. How much of his money is invested at 5% ?
(a) Rs. 5000 (b) Rs. 4225
(c) Rs. 4800 (d) Rs. 3725

33. If the difference between the simple interest and compound interest on some principal amount at 20% per annum for 3 years is Rs. 48, then the principle amount must be
(a) Rs. 550 (b) Rs. 500
(c) Rs. 375 (d) Rs. 400

34. Raju lent Rs. 400 to Ajay for 2 years, and Rs. 100 to Manoj for 4 years and received together from both Rs. 60 as interest. Find the rate of interest, simple interest being calculated.
(a) 5% (b) 6% (c) 8% (d) 9%

35. In what time will Rs. 8000 amount to 40,000 at 4% per annum? (simple interest being reckoned)
(a) 100 years (b) 50 years
(c) 110 years (d) 160 years

36. What annual payment will discharge a debt of Rs. 808 due in 2 years at 2% per annum?
(a) Rs. 200 (b) Rs. 300
(c) Rs. 400 (d) Rs. 350

37. A sum of money becomes 4 times at simple interest in 10 years. What is the rate of interest?
(a) 10% (b) 20% (c) 30% (d) 40%

38. A sum of money doubles itself in 5 years. In how many years will it become four fold (if interest is compounded)?
(a) 15 (b) 10 (c) 20 (d) 12

39. A difference between the interest received from two different banks on Rs. 400 for 2 years is Rs. 4. What is the difference between their rates?
(a) 0.5% (b) 0.2%
(c) 0.23% (d) 0.52%

40. A sum of money placed at compound interest doubles itself in 3 years. In how many years will it amount to 8 times itself?
(a) 9 years (b) 8 years
(c) 27 years (d) 7 years

41. If the compound interest on a certain sum for 2 years is Rs. 21. What could be the simple interest?
(a) Rs. 20 (b) Rs. 16 (c) Rs. 18 (d) Rs. 20.5

42. Divide Rs. 6000 into two parts so that simple interest on the first part for 2 years at 6% p.a. may be equal to the simple interest on the second part for 3 years at 8% p.a.
(a) Rs. 4000, Rs. 2000 (b) Rs. 5000, Rs. 1000
(c) Rs. 3000, Rs. 3000 (d) None of these

43. Divide Rs. 3903 between Amar and Akbar such that Amar's share at the end of 7 years is equal to Akbar's share at the end of 9 years at 4% p.a. rate of compound interest.
(a) Amar = Rs. 2028, Akbar = Rs. 1875
(b) Amar = Rs. 2008, Akbar = Rs. 1000
(c) Amar = Rs. 2902, Akbar = Rs. 1001
(d) Amar = Rs. 2600, Akbar = Rs. 1303

44. A sum of money becomes 7/4 of itself in 6 years at a certain rate of simple interest. Find the rate of interest.
(a) 12% (b) 12(1/2)%
(c) 8% (d) 14%

45. Sanjay borrowed Rs. 900 at 4% p.a. and Rs. 1100 at 5% p.a. for the same duration. He had to pay Rs. 364 in all as interest. What is the time period in years?
(a) 5 years (b) 3 years
(c) 2 years (d) 4 years

increase in the proportion of the soft loan component is only applicable for the first year. For all subsequent years, the soft loan component applicable on the loan, follows the values provided in the table. The widow of a soldier takes Rs. 40,000 under scheme 1 in one account for 1 year and Rs. 60,000 under scheme 2 for 2 years. Find the total interest paid by her over the 2 year period.

(a) Rs. 11,600 (b) Rs. 10,000
(c) Rs. 8800 (d) None of these

29. A sum is divided between *A* and *B* in the ratio of 1 : 2. *A* purchased a car from his part, which depreciates $14\frac{2}{7}$ % per annum and *B* deposited his amount in a bank, which pays him 20% interest per annum compounded annually. By what percentage will the total sum of money increase after two years due to this investment pattern (approximately).

(a) 20% (b) 26.66%
(c) 30% (d) 25%

30. Michael Bolton has $90,000 with him. He purchases a car, a laptop and a flat for $15,000, $13,000 and $35,000 respectively and puts the remaining money in a bank deposit that pays compound interest @15% per annum. After 2 years, he sells off the three items at 80% of their original price and also withdraws his entire money from the bank by closing the account. What is the total change in his asset?

(a) −4.5% (b) +3.5%
(c) −4.32% (d) +5.5%

Hints and Solutions II

3. $x + 0.1x = 1100$
$y + 0.2y = 1800$
6. Interest will be charged on the initial amount borrowed, on the amount of principal still to be paid.
7. Required value = $200000 \times (1.03)^2$
(Solve using percentage change graphic)
9. $(1.105)^3 \times 1000 - 1.3 \times 1000$
12. $64000 \times (1.025)^4$
13. Half-yearly compounding always increases the value of the amount more than annual compounding. Since, increase over 2 years is equal, the annual compounding rate has to be more than the half-yearly compounding rate. Hence $S > R$.
15. Solve through options and use percentage change graphic.
16. If P is the population on 1 January 1995 then
$P \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 0.8 \times 1.1 = 1283575$.
Use options and percentage change graphic to calculate.
20. $8000 \times (1.05)^3 = x \times (1.05)^2 + x \times (1.05) + x$.
24. Solve through options.
27-28. The meaning of the table is
Under Scheme 1 the maximum borrowing allowed is Rs. 50000. The normal interest to be charged is 8% per annum and for the soft loan component 4% per annum has to be charged. It is also given that 50% of the loan taken is the soft loan component.
29. Difference between the total values at the start and at the end is
$$\left(200 \times 1.2 \times 1.2 + 100 \times \frac{6}{7} \times \frac{6}{7}\right) - (200 + 100)$$
30. Final assets = $63000 \times 0.8 + 27000 \times (1.15)^2$
Calculate using percentage change graphic.

Detailed Solutions & Short Cuts

LOD I

1. The annual interest would be Rs. 60. After 3 years the total value would be $1200 + 60 \times 3 = 1380$
4. $1400 \xrightarrow{10\%\uparrow} 1540 \xrightarrow{10\%\uparrow} 1694$.
6. Interest in 2 years = Rs. 240.
Interest per year = Rs. 120
Rate of interest = 10%
7. 12500 @ 10% simple interest would give an interest of Rs. 1250 per annum. For a total interest of Rs. 5000, it would take 4 years.
9. 8% @ 700 = Rs. 56 per year for 3 years
7.5% @ 700 = Rs. 52.5 per year for 2 years
Total interest = $56 \times 3 + 52.5 \times 2 = 273$.
11. Simple interest @ 23% = $4600 \times 2 = 9200$
Compound interest @ 20%
$20000 \xrightarrow{20\%\uparrow} 24000 \xrightarrow{20\%\uparrow} 28800$
→ Rs. 8800 compound interest.
Difference = 9200 − 8800 = Rs. 400.

Interest LOD II

1.	b
2.	c
3.	a
4.	d
5.	b
6.	c
7.	d
8.	b
9.	c
10.	e
11.	a
12.	e
13.	c
14.	a
15.	b
16.	b
17.	b
18.	b
19.	c
20.	a
21.	d
22.	c
23.	b
24.	d
25.	a
26.	c
27.	a
28.	b
29.	a
30.	c

Problem 8.8 Three containers A, B and C are having mixtures of milk and water in the ratio of 1 : 5, 3 : 5 and 5 : 7 respectively. If the capacities of the containers are in the ratio 5 : 4 : 5, find the ratio of milk to water, if the mixtures of all the three containers are mixed together.

Solution Assume that there are 500, 400 and 500 litres respectively in the 3 containers.

Then we have, 83.33,150 and 208.33 litres of milk in each of the three containers.

Thus, the total milk is 441.66 litres. Hence, the amount of water in the mixture is

$$1400 - 441.66 = 958.33 \text{ litres.}$$

Hence, the ratio of milk to water is

441.66 : 958.33 → 53 : 115 (Using division by 0.33333)

The calculation thought process should be:

(441 × 3 + 2) : (958 × 3 + 1) = 1325 : 2875.
Dividing by 25 → 53 : 115.

Level of Difficulty (LOD) I

1. Divide Rs. 1870 into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal.
   (a) 241, 343, 245 (b) 400, 800, 670
   (c) 470, 640, 1160 (d) 420, 600, 850
   (e) None of these
2. Divide Rs. 500 among A, B, C and D so that A and B together get thrice as much as C and D together, B gets four times of what C gets and C gets 1.5 times as much as D. Now the value of what B gets is
   (a) 300 (b) 75
   (c) 125 (d) 150
   (e) None of these
3. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, then each fraction is equal to
   (a) $(a + b + c)^2$ (b) 1/2
   (c) 1/4 (d) 0
   (e) None of these
4. If $6x^2 + 6y^2 = 13xy$, what is the ratio of x to y?
   (a) 2 : 3 (b) 3 : 2 (c) 4 : 5 (d) 1 : 2
   (Hint: Use options to solve fast)
5. If $a : b = c : d$ then the value of $\frac{a^2+b^2}{c^2+d^2}$ is
   (a) 1/2 (b) $\frac{a+b}{c+d}$ (c) $\frac{a-b}{c-d}$ (d) $\frac{ab}{cd}$
   (e) $\frac{ac}{bd}$
6. A crew can row a certain course up the stream in 84 minutes; they can row the same course down stream in 9 minutes less than they can row it in still water. How long would they take to row down with the stream.
   (a) 45 or 23 minutes (b) 63 or 12 minutes
   (c) 60 minutes (d) 19 minutes
   (e) 25 minutes
7. If a, b, c, d are in continued proportion then $\frac{a-d}{b-c} \geq x$. What is the value of x.
   (a) 2 (b) 3 (c) 1 (d) 0
   (e) 4
8. If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day, for how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days.
   (a) 6 (b) 7 ½ (c) 8 (d) 9
   (e) 10
9. If a, b, c, d are proportional, then $(a - b)(a - c)/a =$
   (a) $a + c + d$ (b) $a + d - b - c$
   (c) $a + b + c + d$ (d) $a + c - b - d$
   (e) None of these
10. In a mixture of 40 litres, the ratio of milk and water is 4 : 1. How much water must be added to this mixture so that the ratio of milk and water becomes 2 : 3.
    (a) 20 litres (b) 32 litres
    (c) 40 litres (d) 30 litres
    (e) 35 litres
11. If A varies as C, and B varies as C, then which of the following is false:
    (a) $(A + B) \propto C$ (b) $(A - B) \propto 1/C$
    (c) $\sqrt{AB} \propto C$ (d) $AB \propto C^2$
    (e) None of these
12. If three numbers are in the ratio of 1 : 2 : 3 and half the sum is 18, then the ratio of squares of the numbers is:
    (a) 6 : 12 : 13 (b) 1 : 2 : 4
    (c) 36 : 144 : 324 (d) 3 : 5 : 7
    (e) None of these

So, $S = 42 - 6\sqrt{n}$

For 49 compartments the train would not move. Hence it would move for 48 compartments.

26-28:

Let the third drunkard get in x litres. Then the second will contribute $x + 1$ and the first will contribute $x + 2$ litres. Thus in all they have $3x + 3$ litres of the drink. Using option a in question 27, this value is 12, giving x as 3.

Also, each drunkard will drink 3 litres.

Thus, the first drunkard brings 5 litres and the second 4 litres. Their contribution to the fourth drunkard will be in the ratio 2:1 and hence their share of money would be also in the ratio 2:1. Hence, this option is correct for question 27.

Hence, for question 26, the second drunkard will get 5 roubles (for his contribution of 1 litre to the fourth) and for question 28, the answer would be 1:3

29. 5 : 4 → 5 : 4.8 → 25 : 24.
Option (c) is correct.

31. The ratio of distribution should be:
$21 \times 35 : 15 \times 35 : 15 \times 21 \rightarrow 147 : 105:63 \rightarrow 7:5:3$
The biggest share will be worth: $7 \times 525000/15 = 245000$.

33. Ratio of distribution = 20 : 13 : 8
So the elephant should get $(20/41) \times 820 = 400$.

34. Women : Men = 3 : 4
Men : Children = 3 : 5
→ Women : Men : children = 9 : 12 : 20
In the ratio, 9 → 531 Women
Thus, 20 → 1180 children.

36. Since, the work gets done in 25% less time there must have been an addition of 33.33% men.
This would mean 13.33 men extra → which would mean 14 extra men (in whole nos.)

38. This is a simple question if you can catch hold of the logic of the question. i.e. the younger daughter's share must be such after adding a CI of 20% for two years, she should get the same value as her elder sister.
None of the options meets this requirement. Hence, None of these is correct.

39-41. You should realise that when Anshu gives her pens to Bobby & Chandana, the number of pens for both Bobby & Chandana should double. Also, the number of pens for Anshu & Bobby should also double when Chandana gives off her pens. Further the final condition is that each of them has 24 pens. The following table will emerge on the basis of this logic.

	Anshu	Bobby	Chandana
Final	24	24	24
Second round	12	12	48
Initial	42	6	24

43. Expenses for 120 boys = 8400
Expenses for 150 boys = 10000.
Thus, variable expenses are Rs. 1600 for 30 boys.
If we add 180 more boys to make it 330 boys, we will get an additional expense of $1600 \times 6 =$ Rs. 9600.
Total expenses are Rs. 19600.

45. 47 : 100: 220 would give: 0.5 cubic feet of Cement, 1cubic feet of sand and 2 cubic feet of gravel. Required ratio 1 : 2 : 4 is satisfied.

LOD III

1. You can use alligation between 33.33% and 40% to get 37.5%. Hence the ratio of mixing must be 2.5:4.16 → 3:5

6. Check each of the options as follows:
Suppose you are checking option b which gives the value of a as 81 litres.
Then, it is clear that when you are pouring out 81 litres, you are leaving 8/9 of the honey in the barrel. Thus the amount of honey contained after 6 such operations will be given by:
$729 \times (8/9)^6$. If this answer has to be correct this value must be equal to 64 (which it clearly is not since the value will be in the form of a fraction.)
Hence, this is not the correct option. You can similarly rule out the other options.

7. It is clear that if 7 kg of the first is mixed with 21 kg of the second you will get 5 + 9 = 14 kg of nickel and 14 kg of tin. You do not need to check the other options since they will go into fractions.

10. The piece that is cut off should be such that the fraction of the first to the second alloy in each of the two new alloys formed should be equal.
If you cut off 4 kg, the respective ratios will be:
First alloy: 2 kg of first alloy and 4 kg of second alloy
Second alloy: 4 kg of first alloy and 8 kg of the second alloy. It can easily be seen that the ratios are equal to 1:2 in each case.

Ratio, Proportion and Variation LOD III

1. a
2. a
3. a
4. a
5. a
6. a
7. c
8. b
9. a
10. a
11. c
12. a
13. c
14. b
15. a
16. b
17. b
18. d
19. d
20. c
21. b
22. a
23. d
24. b
25. c

The Specific Case of Building a Wall (Work as Volume of Work)

As already mentioned, in certain cases, the unit of work can also be considered to be in terms of the volume of work. For example, building of a wall of a certain length, breadth and height.

In such cases, the following formula applies:

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{m_1 t_1 d_1}{m_2 t_2 d_2}$$

where L, B and H are respectively the length, breadth and height of the wall to be built, while m, t and d are respectively the number of men, the amount of time per day and the number of days. Further, the suffix 1 is for the first work situation, while the suffix 2 is for the second work situation.

Consider the following problem:

Example: 20 men working 8 hours a day can completely build a wall of length 200 meters, breadth 10 metres and height 20 metres in 10 days. How many days will 25 men working 12 hours a day require to build a wall of length 400 meters, breadth 10 metres and height of 15 metres.

This question can be solved directly by using the formula above

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{m_1 t_1 d_1}{m_2 t_2 d_2}$$

Here,	L_1 is 200 metres	L_2 is 400 metres
	B_1 is 10 metres	B_2 is 10 metres
	H_1 is 20 metres	H_2 is 15 metres
while	m_1 is 20 men	m_2 is 25 men
	d_1 is 10 days	d_2 is unknown
and	t_1 is 8 hours a day	t_2 is 12 hours a day

Then we get $200 \times 10 \times 20/400 \times 10 \times 15 = 20 \times 8 \times 10/25 \times 12 \times d_2$

$\therefore \quad d_2 = 5.333/0.6666 = 8$ days

Alternatively, you can also directly write the equation as follows:

$d_2 = 10 \times (400/200) \times (10/10) \times (20/15) \times (20/25) \times (8/12)$

This can be done by thinking of the problem as follows:

The number of days have to be found out in the second case. Hence, on the LHS of the equation write down the unknown and on the RHS of the equation write down the corresponding knowns.

$$d_2 = 10 \times ....$$

Then, the length of the wall has to be factored in. There are only two options for doing so. viz:

Multiplying by 200/400 (< 1, which will reduce the number of days) or multiplying by 400/200 (>1, which will increase the number of days).

The decision of which one of these is to be done is made on the basis of the fact that when the length of the wall is increasing, the number of days required will also increase.

Hence, we take the value of the fraction greater than 1 to get

$$d_2 = 10 \times (400/200)$$

We continue in the same way to get

No change in the breadth of the wall → hence, multiply by 10/10 (no change in d_2)

Height of the wall is decreasing → hence, multiply by 15/20 (< 1 to reduce d_2)

Number of men working is increasing → hence, multiply by 20/25 (< 1 to reduce d_2)

Number of hours per day is increasing → hence, multiply by 8/12 (< 1 to reduce the number of days)

The Concept of Efficiency

The concept of efficiency is closely related to the concept of work rate.

When we make a statement saying A is twice as efficient as B, we mean to say that A does twice the work as B in the same time. In other words, we can also understand this as A will require half the time required by B to do the same work.

In the context of efficiency, another statement that you might come across is A is two times more efficient than B. This is the same as A is thrice as efficient as B or A does the same work as B in 1/3rd of the time.

Equating Men, Women and Children This is directly derived from the concept of efficiencies.

Example: 8 men can do a work in 12 days while 20 women can do it in 10 days. In how many days can 12 men and 15 women complete the same work.

Solution: Total work to be done = 8 × 12 = 96 man-days.

or total work to be done = 20 × 10 = 200 woman-days.

Since, the work is the same, we can equate 96 man-days = 200 woman-days.

Hence, 1 man-day = 2.08333 woman-days.

100 Sepoys can hold 5% of the enemy for one month.

100 Mantris can hold 10% of the enemy for 15 days.

50 Footies can hold 5% of the enemy for one month.

A sepoy eats 1 kg of food per month, a Mantri eats 0.5 kg of food per month and a footie eats 3 kg of food. (Assume 1 ton = 1000 kg).

The king has to make some decisions based on the longest possible resistance that can be offered to the enemy.

If a king selects a soldier, he will have to feed him for the entire period of the resistance. The king is not obliged to feed a soldier not selected for the resistance.

(Assume that the entire food allocated to a particular soldier for the estimated length of the resistance is redistributed into the king's palace in case a soldier dies and is not available for the other soldiers.)

11. If the king wants to maximise the time for which his resistance holds up, he should
    (a) Select all mantris (b) Select all footies
    (c) Select all sepoys (d) None of these
12. Based on existing resources, the maximum number of months for which the fort's resistance can last is
    (a) 5 months (b) 20 months
    (c) 7.5 months (d) Cannot be determined
13. If the king makes a decision error, the maximum reduction in the time of resistance could be
    (a) 15 months (b) 12.5 months
    (c) 16.66 months (d) Cannot be determined
14. If the king estimates that the attackers can last for only 50 months, what should the king do to ensure victory?
    (a) Select all mantris
    (b) Select the mantris and the sepoys
    (c) Select the footies
    (d) The king cannot achieve this
15. If a reduction in the ration allocation by 10% reduces the capacity of any soldier to hold off the enemy by 10%, the number of whole months by which the king can increase the life of the resistance by reducing the ration allocation by 10% is
    (a) 4 months (b) 2 months
    (c) No change (d) This will reduce the time
16. The minimum amount of grain that should be available in the granary to ensure that the fort is not lost (assuming the estimate of the king of 50 months being the duration for which the enemy can last is correct) is
    (a) 2000 tons (b) 2500 tons
    (c) 5000 tons (d) Cannot be determined
17. If the king made the worst possible selection of his soldiers to offer the resistance, the percentage increase in the minimum amount of grain that should be available in the granary to ensure that the fort is not lost is
    (a) 100% (b) 500%
    (c) 600% (d) Cannot be determined
18. The difference in the minimum grain required for the second worst choice and the worst choice to ensure that the resistance lasts for 50 months is
    (a) 5000 tons (b) 7500 tons
    (c) 10000 tons (d) Cannot be determined
19. If the king strategically attacks the feeder line on the first day of the resistance so that the grain is no longer a constraint, the maximum time for which the resistance can last is
    (a) 100 months (b) 150 months
    (c) 250 months (d) Cannot be determined
20. If the feeder line is opened after 6 months and prior to that the king had made decisions based on food availability being a constraint then the number of months (maximum) for which the resistance could last is
    (a) 100 months (b) 150 months
    (c) 5 months (d) Cannot be determined

Directions for Questions 21–25: Study the following and answer the questions that follow.

A gas cylinder can discharge gas at the rate of 1 cc/minute from burner *A* and at the rate of 2 cc/minute from burner *B* (maximum rates of discharge). The capacity of the gas cylinder is 1000 cc of gas.

The amount of heat generated is equal to 1 kcal per cc of gas.

However, there is wastage of the heat as per follows:

Gas discharge@	*Loss of heat*
0–0.5 cc/minute	10%
0.5–1 cc/minute	20%
1–1.5 cc/minute	25%
1.5 + cc/minute	30%

@(Include higher extremes)

21. If both burners are opened simultaneously such that the first is opened to 90% of its capacity and the second is opened to 80% of its capacity, the amount of

Time and Work LOD II

1.	a
2.	b
3.	d
4.	c
5.	d
6.	b
7.	e
8.	a
9.	b
10.	d
11.	c
12.	b
13.	b
14.	b
15.	c
16.	b
17.	a
18.	a
19.	a
20.	b
21.	c
22.	d
23.	c
24.	a
25.	a
26.	c
27.	b
28.	c
29.	a
30.	b

Hence, speed ratio = distance ratio
→ 4/5 = distance ratio

Hence, the meeting point will be 400 km from A and 500 km from B.

3. Inverse proportionality between speed and time (when the distance is constant) Speed ∝ 1/time

(a) *A body travels at S_1 kmph for the first half of the journey and then travels at S_2 kmph for the second half of the journey.* Here two motions of one body are being described and between these two motions the distance travelled is constant. Hence the speed will be inversely proportional to the time travelled for.

(b) Two cars start simultaneously from A and B respectively towards each other. They meet at a point C and reach their respective destinations B and A in t_1 and t_2 hours respectively... Here again, the speed is inversely proportional to the time since two motions are described where the distance of both the motions is the same, that is, it is evident here that the first and the second car travel for the distance, viz., AB.
In such a case, the following ratio will be valid:

$$S_1/S_2 = t_2/t_1 \quad \text{i.e.} \quad S_1t_1 = S_2t_2 = S_3t_3 = K$$

Illustrations

(i) A train meets with an accident and moves at 3/4 its original speed. Due to this, it is 20 minutes late. Find the original time for the journey beyond the point of accident.

Solution: Speed becomes 3/4 (Time becomes 4/3)
Extra time = 1/3 of normal time = 20 minutes
Normal time = 60 minutes

Alternatively, from the table on product constancy in the chapter of percentages, we get that a 25% reduction in speed leads to a 33.33% increase in time.

But, 33.33% increase in time is equal to 20 minutes increase in time.
Hence, total time (original) = 60 minutes.

(ii) A body travels half the journey at 20 kmph and the other half at 30 kmph. Find the average speed.

Solution: The short-cut process is elucidated in the chapter on 'averages'. Answer = 24 kmph.

(iii) A man travels from his house to his office at 5 km/h and reaches his office 20 minutes late. If his speed had been 7.5 km/h, he would have reached his office 12 minutes early. Find the distance from his house to his office.

Solution: Notice that here the distance is constant. Hence, speed is inversely proportional to time.
Solving mathematically

$$S_1/S_2 = t_2/(t_2 + 32)$$
$$5/7.5 = t_2/(t_2 + 32)$$
$$5t_2 + 160 = 7.5\ t_2$$
$$t_2 = 160/2.5 = 64 \text{ minutes}$$

Hence, the distance is given by 7.5 × 64/60 = 8 km.

Alternatively, using the Product Constancy Table from the chapter of percentages. If speed increases by 50%, then time will decrease by 33.33%.

But the decrease is equal to 32 minutes.
Hence, original time = 96 minutes and new time is 64 minutes.
Hence, the required distance = 5 × 96/60 km = 8 km.
or distance = 7.5 × 64/60 km = 8 km

[*Note:* The entire process can be worked out mentally while reading the problem.]

CONVERSION BETWEEN kmph to m/s

1 km/h = 1000 m/h = 1000/3600 m/s = 5/18 m/s.

Hence, to convert y km/h into m/s multiply by 5/18.

Thus, y km/h = $\frac{5y}{18}$ m/s.

And vice versa : y m/s = 18 y/5 km/h. To convert from m/s to kmph, multiply by 18/5.

Relative Speed : Same Direction and Opposite Direction

Normally, when we talk about the movement of a body, we mean the movement of the body with respect to a stationary point. However, there are times when we need to determine the movement and its relationships with respect to a moving point/body. In such instances, we have to take into account the movement of the body/point with respect to which we are trying to determine relative motion.

Relative movement, therefore, can be viewed as the movement of one body relative to another moving body.

Aurangabad. Find the speed of the cyclist who started from Ellora.

(a) 12 km/h (b) 16 km/h
(c) 15 km/h (d) 10 km/h
(e) None of these

8. Two ants start simultaneously from two ant holes towards each other. The first ant coveres 8% of the distance between the two ant holes in 3 hours, the second ant covered $\frac{7}{120}$ of the distance in 2 hours 30 minutes. Find the speed (feet/h)of the second ant if the first ant travelled 800 feet to the meeting point.

(a) 15 feet/h (b) 25 feet/h
(c) 45 feet/h (d) 35 feet/h
(e) 36 feet/h

9. A bus left point X for point Y. Two hours later a car left point X for Y and arrived at Y at the same time as the bus. If the car and the bus left simultaneously from the opposite ends X and Y towards each other, they would meet 1.33 hours after the start. How much time did it take the bus to travel from X to Y?

(a) 2 h (b) 4 h (c) 6 h (d) 8 h
(e) 10 h

10. A racetrack is in the form of a right triangle. The longer of the legs of the track is 2 km more than the shorter of the legs (both these legs being on a highway). The start and end points are also connected to each other through a side road. The escort vehicle for the race took the side road and rode with a speed of 30 km/h and then covered the two intervals along the highway during the same time with a speed of 42 km/h. Find the length of the racetrack.

(a) 14 km (b) 10 km
(c) 24 km (d) 36 km
(e) 30 km

11. Two planes move along a circle of circumference 1.2 km with constant speeds. When they move in different directions, they meet every 15 seconds and when they move in the same direction, one plane overtakes the other every 60 seconds. Find the speed of the slower plane.

(a) 0.04 km/s (b) 0.03 km/s
(c) 0.05 km/s (d) 0.02 km/s
(e) None of these

12. Karim, a tourist leaves Ellora on a bicycle. Having travelled for 1.5 h at 16 km/h, he makes a stop for 1.5 h and then pedals on with the same speed. Four hours after Karim started, his friend and local guide Rahim leaves Ellora on a motorcycle and rides with a speed of 28 km/h in the same direction as Karim had gone. What distance will they cover before Rahim overtakes Karim.

(a) 88 km (b) 90.33 km
(c) 93.33 km (d) 96.66 km
(e) None of these

13. A tourist covered a journey partly by foot and partly by tonga. He walked for 90 km and rode the tonga for 10 km. He spent 4 h less on the tonga than on walking. If the tourist had reversed the times he travelled by foot and on tonga, the distances travelled on each part of the journey would be equal. How long did he ride the tonga?

(a) He rode for 6 hours (b) He rode for 4 hours
(c) He rode for 2 hours (d) He rode for 5 hours
(e) None of these

14. Two Indian tourists in the US cycled towards each other, one from point A and the other from point B. The first tourist left point A 6 hrs later than the second left point B, and it turned out on their meeting that he had travelled 12 km less than the second tourist. After

second arrived at A

faster tourist.

(e) 5 km/h

when he was 2 km short of Noida. If the first jogger jogged as many kilometres as the second and the second as many kilometres as the first, the first one would need 17 min less than the second. Find the distance between Delhi and Noida?

(a) 5 km (b) 15 km (c) 25 km (d) 35 km
(e) 24 km

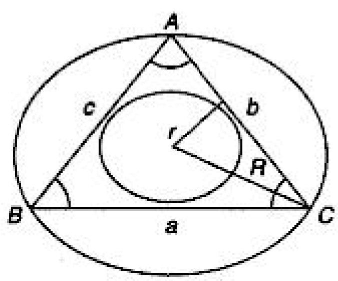
16. A tank of 4800 m^3 capacity is full of water. The discharging capacity of the pump is 10 m^3/min higher than its filling capacity. As a result the pump needs 16 min less to discharge the fuel than to fill up the tank. Find the filling capacity of the pump.

6. *Scalene Triangle:* Triangle with none of the sides equal to any other side.

Properties (General)

- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: $a/\sin A = b/\sin B = c/\sin C = 2R$ (where R = circum radius.)
- The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

  This is true for all sides and respective angles.

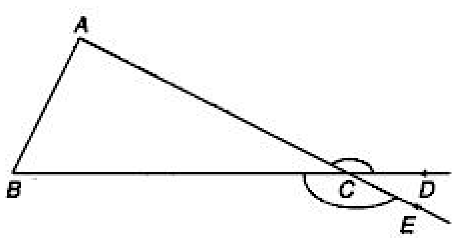


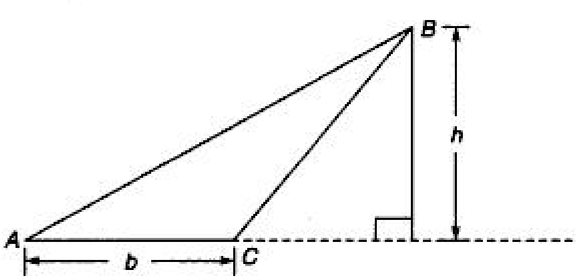
In case of a right ∠, the formula reduces to $a^2 = b^2 + c^2$

Since cos 90 = 0

- The exterior angle is equal to the sum of two interior angles not adjacent to it.

$$\angle ACD = \angle BCE = \angle A + \angle B$$





Area

1. Area = 1/2 base × height or 1/2 bh.

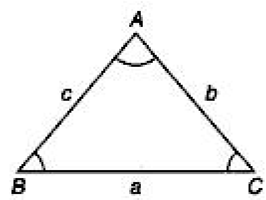
   Height = Perpendicular distance between the base and vertex opposite to it
2. Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Hero's formula)

   where $S = \dfrac{a+b+c}{2}$ (a, b and c being the length of the sides)
3. Area = rs (where r is in radius)
4. Area = 1/2 × product of two sides × sine of the included angle

   = 1/2 ac sin B

   = 1/2 ab sin C

   = 1/2 bc sin A



4. Area = $abc/4R$

   where R = circum radius

Congruency of Triangles Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is $(\cong)$.

Conditions for Congruency

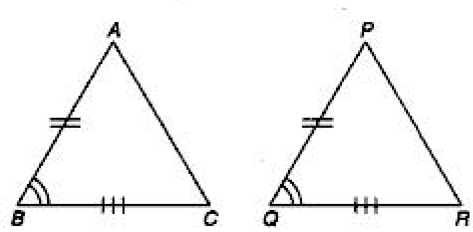
1. *SAS congruency*: If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent. (See figure below.)

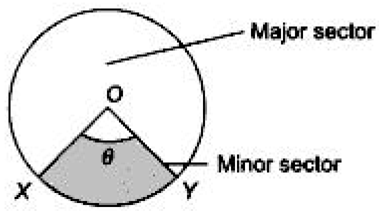
Here, $AB = PQ$

$BC = QR$

and $\angle B = \angle Q$

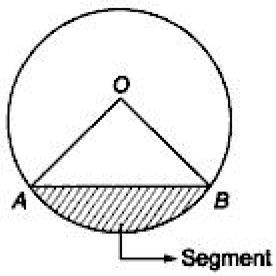
So $\Delta ABC \cong \Delta PQR$





(g) **Segment:** A sector minus the triangle formed by the two radii is called the segment of the circle.

(h) Area = Area of the sector − Area ΔOAB = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$



(i) Perimeter of segment = length of the arc + length of segment AB

$$= \frac{\theta}{360} \times 2\pi r + 2r \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{\pi r\theta}{180} + 2r \sin\left(\frac{\theta}{2}\right)$$

(j) **Congruency:** Two circles can be congruent if and only if they have equal radii.

Properties

(a) The perpendicular from the centre of a circle to a chord bisects the chord. The converse is also true.

(b) The perpendicular bisectors of two chords of a circle intersect at its centre.

(c) There can be one and only one circle passing through three or more non-collinear points.

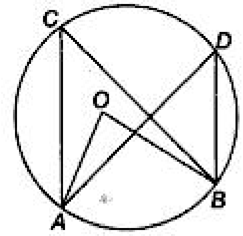
(d) If two circles intersect in two points then the line through the centres is the perpendicular bisector of the common chord.

(e) If two chords of a circle are equal, then the centre of the circle lies on the angle bisector of the two chords.

(f) Equal chords of a circle or congruent circles are equidistant from the centre.

(g) Equidistant chords from the centre of a circle are equal to each other in terms of their length.

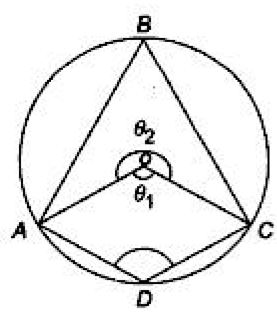
(h) The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle. This can be clearly seen in the following figure:
With respect to the arc AB, $\angle AOB = 2\angle ACB$.



(i) Any two angles in the same segment are equal. Thus, $\angle ACB = \angle ADB$.

(j) The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

(k) Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.
In the figure below



$\angle ABC$ is acute and

$$\angle ADC = \text{obtuse}$$

Also $\quad \theta_1 = 2\angle B$

And $\quad \theta_2 = 2\angle D$

$\therefore \quad \theta_1 + \theta_2 = 2(\angle B + \angle D)$

$\qquad = 360° = 2(\angle B + \angle D)$

or $\quad \angle B + \angle D = 180°$

or sum of opposite angles of a cyclic quadrilateral is 180°.

(l) If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic. Thus, in the following figure:

Height of a hemisphere = Radius of its base

So the question is effectively asking us to find out h/r

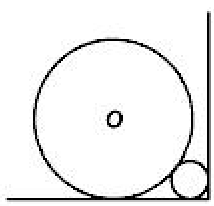
By the formula above we can easily see that $h/r = 2/1$

HOW TO THINK IN GEOMETRY AND MENSURATION

In the back to school section of this block, we have already mentioned that there is very little use of complex and obscure formulae and results while solving questions on this block.

The following is a list of questions (with solutions) of what has been asked in CAT over the past few years. Hopefully you will realise through this exercise, what we are talking about when we say this. For each of the questions given below, try to solve on your own first, before looking at the solution provided.

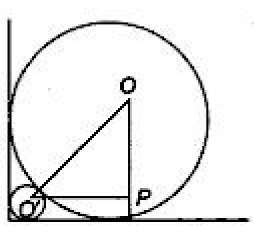
1. A circle with radius 2 is placed against a right angle. As shown in the figure below, another smaller circle is placed in the gap between the circle and the right angle. What is the radius of the smaller circle?



(a) $3-2\sqrt{2}$ (b) $4-2\sqrt{2}$

(c) $7-4\sqrt{2}$ (d) $6-4\sqrt{2}$

Solution: The solution of the above question is based on the following construction.



In the right triangle $OO'P$,

$$OP = (2-r),\ O'P = (2-r) \text{ and } OO' = 2 + r$$

where r is the radius of the smaller circle.

Using Pythagoras theorem:

$$(2+r)^2 = (2-r)^2 + (2-r)^2 + (2-r)^2$$

Solving, we get $r = 6 \pm 4\sqrt{2}$

$6 + 4\sqrt{2}$ cannot be correct since the value of r should be less than 2.

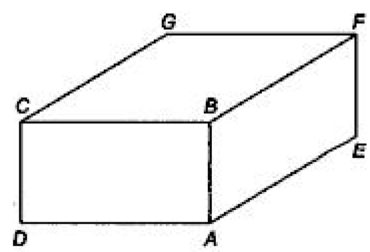
Note: The key to solving this question is in the visualisation of the construction. If you try to use complex formulae while solving, your mind unnecessarily gets cluttered. The key to your thinking in this question is:

(1) Realise that you only have to use length measuring formulae. Hence, put all angle measurement formulae into the back seat.

(2) A quick mental search of the length measuring formulae available for this situation will narrow down your mind to the Pythagoras theorem.

(3) The key then becomes the construction of a triangle (right angled of course) where the only unknown is r.

2. ABCDEFGH is a cube. If the length of the diagonals DF, AG & CE are equal to the sides of a triangle, then the circumradius of that triangle would be



(a) Equal to the side of the cube

(b) $\sqrt{3}$ times the side of the cube

(c) $1/\sqrt{3}$ times the side of the cube

(d) Indeterminate

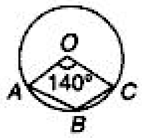
Solution: If we assume the side of the cube to be a the triangle will be an equilateral triangle with side $a\sqrt{3}$. (we get this using Pythagoras theorem). Also, we know that the circumradius of an equilateral triangle is $1/\sqrt{3}$ times the side of the triangle.

Hence, in this case the circumradius would be a—equal to the side of the cube.

(Again the only formula used in this question would be the Pythagoras theorem.)

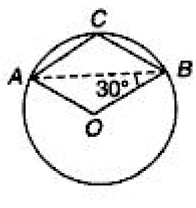
3. On a semicircle (diameter AD), chord BC is parallel to the diameter AD. Also, $AB = CD = 2$, while $AD = 8$, what is the length of BC.

20. In the following figure, it is given that O is the centre of the circle and $\angle AOC = 140°$. Find $\angle ABC$.



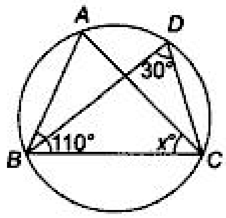
(a) 110° (b) 120° (c) 115°
(d) 130° (e) 100°

21. In the following figure, O is the centre of the circle and $\angle ABO = 30°$, find $\angle ACB$.



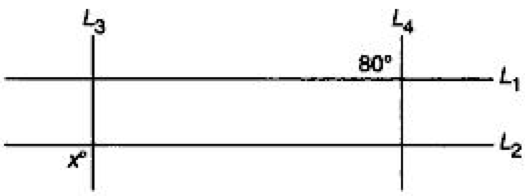
(a) 60° (b) 120°
(c) 75° (d) 90°
(e) 110°

22. In the following figure, find the value of x



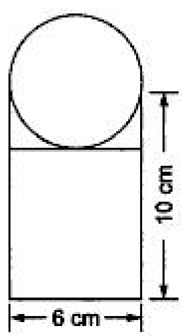
(a) 40° (b) 25°
(c) 30° (d) 45°
(e) 50°

23. If $L_1 \parallel L_2$ in the figure below, what is the value of x.



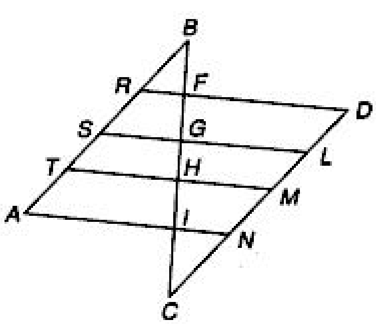
(a) 80° (b) 100°
(c) 40° (d) 90°
(e) Cannot be determined

24. Find the perimeter of the given figure.



(a) $(32 + 3\pi)$ cm
(b) $(36 + 6\pi)$ cm
(c) $(46 + 3\pi)$ cm
(d) $(26 + 6\pi)$ cm
(e) $(26 + 3\pi)$ cm

25. In the figure, AB is parallel to CD and $RD \parallel SL \parallel TM \parallel AN$, and $BR : RS : ST : TA = 3 : 5 : 2 : 7$. If it is known that $CN = 1.333\ BR$. find the ratio of $BF : FG : GH : HI : IC$



(a) 3 : 7 : 2 : 5 : 4 (b) 3 : 5 : 2 : 7 : 4
(c) 4 : 7 : 2 : 5 : 3 (d) 4 : 5 : 2 : 7 : 3
(e) None of these

MENSURATION

Level of Difficulty (LOD) I

1. In a right angled triangle, find the hypotenuse if base and perpendicular are respectively 36015 cm and 48020 cm.
   (a) 69125 cm (b) 60025 cm
   (c) 391025 cm (d) 60125 cm
   (e) None of these
2. The perimeter of an equilateral triangle is $72\sqrt{3}$ cm. Find its height.
   (a) 63 metres (b) 24 metres
   (c) 18 metres (d) 36 metres
   (e) 32 metres
3. The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the diameter of the outer circle of the track.
   (a) 84 cm (b) 168 cm
   (c) 336 cm (d) 77 cm
   (e) 80 cm
4. A race track is in the form of a ring whose inner and outer circumference are 352 metre and 396 metre respectively. Find the width of the track.
   (a) 7 metres (b) 14 metres
   (c) 14π metres (d) 7π metres
   (e) None of these
5. The outer circumference of a circular track is 220 metre. The track is 7 metre wide everywhere. Calculate the cost of levelling the track at the rate of 50 paise per square metre.
   (a) Rs. 1556.5 (b) Rs. 3113
   (c) Rs. 593 (d) Rs. 1386
   (e) Rs. 693
6. Find the area of a quadrant of a circle whose circumference is 44 cm
   (a) 77 cm^2 (b) 38.5 cm^2
   (c) 19.25 cm^2 (d) 19.25π cm^2
   (e) None of these

LOD II

1. For the function to be defined $4 - x^2 > 0$
   This happens when $-2 < x < 2$.
   Option (a) is correct.
2. For the function to be defined two things should happen
   (a) $(1 - x) > 0 \Rightarrow x < 1$ and
   (b) $(x + 2) \geq 0 \Rightarrow x \geq -2$. Also $x \neq 0$
   Thus, option (d) is correct.
3. $\frac{5x - x^2}{4} \geq 1 \Rightarrow 1 \leq x \leq 4$.
4. Neither 2^{-x+} nor $2^{x-x+x+x}$ is an odd function as for neither of them is $f(x) = -f(-x)$
5. $1 - |x|$ should be non negative.
   $[-1, 1]$ would satisfy this.
6. $4 - x^2 \neq 0$ and $(x^3 - x) > 0 \Rightarrow (-1, 0) \cup (1, \infty)$ but not 2 or -2.
7. $f(0) = 1, f(1) = 2$ and $f(2) = 4$
   Hence, they are in G.P.
8. x would become -2 and $y = -3$.
9. $4(f(v(t))) = a\,(f(t^2)) = 4\,(1/t^2) = (4/t^2) - 5$
10. $g(f(h(t))) = g\,(f(4t - 8)) = g\left(\sqrt{4t-8}\right)$
    $= \frac{\sqrt{4t-8}}{4}$
11. $h(g(f(t))) = h\,(g(\sqrt{t})) = h\,(\sqrt{t}/4)$
    $= \sqrt{t} - 8$
12. $f(h(g(t))) = f(h(t/4)) = f(t - 8) = \sqrt{t-8}$.
13. All three functions would give the same values for $x > 0$. As $g(x)$ is not defined for negative x, and $h(x)$ is not defined for $x = 0$.
14. $e^x + e^{-x} = e^{-x} + e^x$
    Hence, this is an even function.
15. $(x + 3)^3$ would be shifted 3 units to the left and hence $(x + 3)^3 + 1$ would shift 3 units to the left and 1 unit up. Option (c) is correct.
16. $f(x) \cdot g(x) = 15x^8$ which is an even function. Thus, option (a) is correct.
17. $(x^2 + \log_e x)$ would be neither odd nor even since it obeys neither of the rules for even function $(f(x) = f(-x))$ nor for odd functions $(f(x) = -f(-x))$.
18. $(x^3 - x^2/5) = f(x) - g(x)$ is neither even nor odd.
19. $y = 1/(x - 2) \Rightarrow (x - 2) = 1/y \;\Rightarrow\; x = 1/y + 2$.
    Hence, $f^{-1}(x) = 1/x + 2$.
20. $y = e^x$
    $\Rightarrow \log_e y = x$.
    $\Rightarrow f^{-1}(x) = \log_e x$.
21. $y = x/(x - 1)$
    $\Rightarrow \quad (x - 1)/x = 1/y$
    $\Rightarrow \quad 1 - (1/x) = 1/y$
    $\Rightarrow \quad 1/x = 1 - 1/y \Rightarrow 1/x = (y - 1)/y$
    $\Rightarrow \quad x = y/(y - 1)$
    Hence, $f^{-1}(x) = x/(x - 1)$.
22. If you differentiate each function with respect to x, and equate it to 0 you would see that for none of the three options will get you a value of $x = -3$ as its solution. Thus, option (d) viz. x one of these is correct.

Directions for Questions 23–32: You essentially have to mark (a) if it is an even function,
mark (b) if it is an odd function,
mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between. This is seen in Questions 25, 26, 30 and 32.

We see even functions in Questions 24 and 28. [Symmetry about the y axis]. We see odd functions in questions 23 and 31.

While the figures in Questions 27 and 29 are neither odd nor even.

Even $\Rightarrow$ 24, 28,
Odd 23, 31.
Neither 27, 29,
doesn't exist: 26, 30, 32.

33. $-f(x)$ would be the mirror image of the function, about the 'x' axis which is seen in option (b).
34. $-f(x) + 1$ would be mirror image about the x axis and then shifted up by 1. Option (a) satisfies this.
35. $f(x) - 1$ would shift down by 1 unit. Thus option (c) is correct.
36. $f(x) + 1$would shift by 1 unit. Thus, option (d) is correct.
37. The given function would become $h[11, 80, 1] = 2640$.
38. The given function would become $g[0, 0, 3] = 0$.
39. The given function would become $f[3, 3, 3] = 27$.
40. $f(1, 2, 3) - g(1, 2, 3) + h(1, 2, 3) = 11 - 23 + 18 = 6$.
41. The number of g's and f's should be equal on the LHS and RHS since both these functions are essentially inverse of each other.
    Option (c) is correct.

- If $a > b$, then evidently $b < a$; that is,

  if the sides of an inequality be transposed, the sign of inequality must be reversed.

- If $a > b$, then $a - b$ is positive, and $b - a$ is negative; that is, $-a-(-b)$ is negative, and therefore $-a < -b$; hence,

  if the signs of all the terms of an inequality be changed, the sign of inequality must be reversed.

  Again, if $a > b$, then $-a < -b$ and, therefore, $-ac < -bc$; that is,

  if the sides of an inequality be multiplied by the same negative quantity, the sign of inequality must be reversed.

  If $a_1 > b_1, a_2 > b_2, a_3 > b_3, \ldots a_m > b_m$, it is clear that $a_1 + a_2 + a_3 + \ldots + a_m > b_1 + b_2 + b_3 + \ldots + b_m$; and $a_1 a_2 a_3 \ldots a_m > b_1 b_2 b_3 \ldots b_m$.

- If $a > b$, and if p, q are positive integers, then or $a^{1/q} > b^{1/q}$ and, therefore, $a^{p/q} > b^{p/q}$; that is, $a^n > b^n$, where n is any positive quantity. Further,

$$1/a^n < 1/b^n; \text{ that is } a^{-n} < b^{-n}$$

  The square of every real quantity is positive, and therefore greater than zero. Thus $(a - b)^2$ is positive.

  Let a and b be two positive quantities, S their sum and P their product. Then from the identity

$$4ab = (a + b)^2 - (a - b)^2$$

  we have $4P = S^2 - (a - b)^2$, and $S^2 = 4P + (a - b)^2$

  Hence, if S is given, P is greatest when $a = b$; and if P is given, S is least when $a = b$;
  That is, *if the sum of two positive quantities is given, their product is greatest when they are equal; and if the product of two positive quantities is given, their sum is least when they are equal.*

To Find the Greatest Value of a Product, the Sum of Whose Factors is Constant

Let there be n factors $a, b, c, \ldots n$, of a composite number and suppose that their sum is constant and equal to S.

Consider the product $abc \ldots n$, and suppose that a and b are any two unequal factors. If we replace the two unequal factors a and b by the two equal factors $(a + b)/2$, and $(a + b)/2$, the product is increased while the sum remains unaltered. Hence, so *long as the product contains two unequal factors it can be increased altering the sum of the factors; therefore, the product is greatest when all the factors are equal.* In this case the value of each of the n factors is S/n, and the greatest value of the product is $(S/n)^n$, or $\{(a + b + c + \ldots + n/n\}^n$

This will be clearer through an example.

Let us define a number as $a \times b = c$ such that we restrict $a + b = 100$ (maximum).

Then, the maximum value of the product will be achieved if we take the value of a and b as 50 each.

Thus $50 \times 50 = 2500$ will be the highest number achieved for the restriction $a + b \leq 100$.

Further, you can also say that $50 \times 50 > 51 \times 49 > 52 \times 48 > 53 \times 47 > 54 \times 46 > \ldots > 98 \times 2 > 99 \times 1$

Thus if we have a larger multiplication as

$4 \times 6 \times 7 \times 8$ this will always be less than $5 \times 5 \times 7 \times 8$. [Holds true only for positive numbers.]

Corollary If $a, b, c \ldots k$, are unequal, $\{(a + b + c + \ldots + k)/n\}^n > abc \ldots k$;
that is, $(a + b + c + \ldots + k)/n > (abc \ldots k)^{1/n}$.

By an extension of the meaning of the arithmetic and geometric means this result is usually quoted as follows: *The arithmetic mean of any number of positive quantities is greater than the geometric mean.*

Definition of Solution of an Inequality

The solution of an inequality is the value of an unknown for which this inequality reduces to a true numerical identity. That is, to solve an inequality means to find all the values of the variable for which the given inequality is true.

An inequality has no solution if there is no such value for which the given inequality is true.

Equivalent Inequalities: Two inequalities are said to be equivalent if any solution of one is also a solution of the other and vice versa.

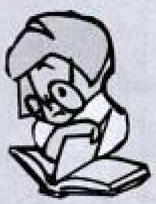
If both inequalities have no solution, then they are also regarded to be equivalent.

To solve an inequality we use the basic properties of an inequality which have been illustrated above.

BLOCK 6

CHAPTERS

- **Permutations and Combinations**
- **Probability**



...BACK TO SCHOOL

In my experience, students can be divided into two broad categories on the basis of their ability in solving chapters in this block.

Category 1: Students who are comfortable in solving questions on this block, since they understand the underlying concepts well.

Category 2: Students who are not able to tackle questions in this block of chapters since they are not conversant with the counting tools and methods in this block.

If you belong to the second category of students, the main thing you would need to do is to familiarize yourself with the counting methods and techniques of Permutations and Combinations. Once you are through with the same, you would find yourself relatively comfortable at both Permutations & Combinations (P&C) and Probability - the chapters in this block. However, before we start looking at these counting methods, right at the outset, I would want you to remove any negative experiences you might have had while trying to study P& C and Probability. So if you belong to the second category of students, you are advised to read on:

Look at the following table:

Suppose, I were to ask you to count the number of cells in the table above, how would you do it??

$$5 \times 5 = 25!!$$

Contd.

Review Test Two

1.	c
2.	c
3.	b
4.	b
5.	b
6.	a
7.	c
8.	c
9.	c
10.	c
11.	a
12.	c
13.	a
14.	d
15.	d
16.	b
17.	b
18.	b
19.	b
20.	b

$A \neq B$, that is, all the elements of set A are not included in set B and all the elements of set B are not included in set A.

Example: $A = \{a, b, c\}$ and $B = \{c, b, a\}$ are equal sets.

Hence, in this case we can write Set A = Set B or simply $A = B$.

6. Subsets Set A is said to be the subset of another set B if all the elements of set A are included in set B.

'Set A is a subset of set B' is shown by $A \subseteq B$.

We can say now that every element of set A is a member of set B.

Example: If $A = \{a, b, c\}$ and $B = \{a, b, c, d, e\}$ then $A \subseteq B$, or A is a subset of B.

Some important results on subsets:

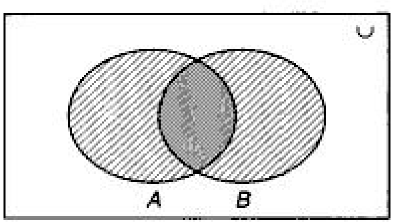
1. Every set is a subset of itself.
2. Every set has an empty set as its subset.
3. Total number of subsets of a set having n elements is 2^n.

7. Universal Set A set which contains all the sets in a given context is a Universal set.

Example: When we are using sets containing natural numbers, N is the universal set.

If $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{c, d\}$

Then we can take $U = \{a, b, c, d\}$ as universal set.



8. Power Set The collection of all the subsets of a set is known as the power set of that set.

If A is the set, then a set containing all the subsets of the A is known as the Power set of A. It is denoted by $P(A)$.

Let $A = \{1, 2\}$, then the number of subsets of this set will be 2^2 and the subsets are $\{\}$, $\{1\}$, $\{2\}$ and $\{1, 2\}$ and the set containing all these four sets is known as Power set represented as $P(A)$.

9. Venn Diagrams Swiss mathematician Euler first gave the idea of representing sets by diagrams. Later on, British mathematician Venn brought this into practice. So, it is known as Euler-Venn diagram or simply Venn diagram. In this way of representing sets, we use a closed curve, generally a circle, to denote sets and their operations.

Operations on Sets

1. Union of Sets If two sets are A and B, then union of A and B is defined as the set that have all the elements which belong to either A or B or both.

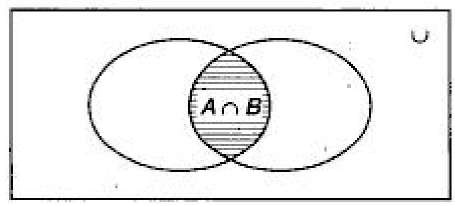
It is represented by $A \cup B$.

2. Intersection of Sets If two sets are A and B, then intersection of A and B is defined as the set that have all the elements which belong to both A and B.

It is represented by $A \cap B$.

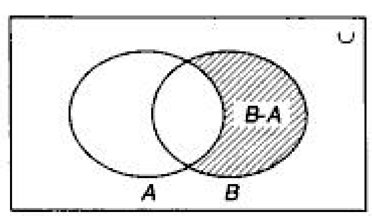
Example: Find $A \cup B$ and $A \cap B$ if $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$
$$A \cap B = \{2, 4\}$$



3. Disjoint Sets Two sets are said to be disjoint if $A \cap B = 0$, that is, not a single element is common to both of these two sets.

Example: If A = {Set of all odd numbers} and B = {Set of all even numbers} then set A and set B are Disjoint sets.



4. Difference of Sets For two sets A and B, $A - B$ is the set of all those elements of A that do not belong to B.

Similarly, $B - A$ is the set of all those elements of B that do not belong to A.

Example: $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 8\}$, then $A - B = \{1, 3, 5, 6\}$ and $B - A = \{8\}$

Illustration

Question 5: Find the in-centre of the right angled isosceles triangle having one vertex at the origin and having the other two vertices at (6, 0) and (0, 6).

Answer: Obviously, the length of the two sides *AB* and *BC* of the triangle is 6 units and the length of the third side is $(6^2 + 6^2)^{1/2}$.

Hence $a = c = 6,\ b = 6\sqrt{2}$

In-centre will be at

$$\frac{(6.0 + 6\sqrt{2}.0 + 6.6)}{(6 + 6 + \sqrt{2})}, \frac{(6.6 + 6\sqrt{2}.0 + 6.0)}{(6 + 6 + \sqrt{2})}$$

$$= \frac{36}{12 + 6\sqrt{2}}, \frac{36}{12 + 6\sqrt{2}}$$

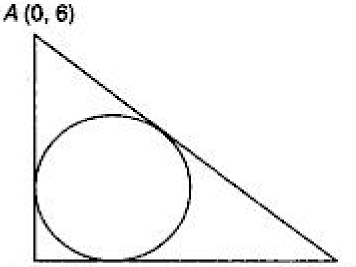


Fig. 19.6

6. Circumcentre of a Triangle The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circumcentre. It is equidistant from the vertices of the triangle. It is also known as the centre of the circle which passes through the three vertices of a triangle (or the centre of the circle that circumscribes the triangle.)

Let *ABC* be a triangle. If *O* is the circumcentre of the triangle *ABC*, then *OA* = *OB* = *OC* and each of these three represent the circum radius.

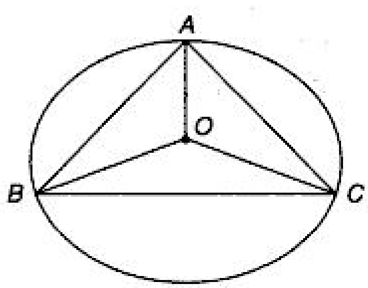


Fig. 19.7

Illustration

Question 6: What will be the circumcentre of a triangle whose sides are $3x - y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$?

Answer: Let *ABC* be the triangle whose sides *AB*, *BC* and *CA* have the equations $3x - y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$ respectively.

Solving the equations, we get the points *A*, *B* and *C* as (–2, –3), (–1, 0) and (7, – 6) respectively.

The equation of a line perpendicular to *BC* is $4x - 3y + k = 0$.

[For students unaware of this formula, read the section on straight lines later in the chapter.]

This will pass through (3, –3), the mid-point of *BC*, if $12 + 9 + k = 0 \Rightarrow k = -21$

Putting $k_1 = -21$ in $4x - 3y + k = 0$, we get $4x - 3y - 21 = 0$ (i)

as the equation of the perpendicular bisector of *BC*.

Again, the equation of a line perpendicular to *CA* is $3x - y + k_1 = 0$.

This will pass through (5/2, –9/2), the mid-point of *AC* if

$$15/2 + 9/2 + k_1 = 0 \Rightarrow k_1 = -12$$

Putting $k_1 = -12$ in $3x - y + k_1 = 0$, we get $3x - y - 12 = 0$ (ii)

as the perpendicular bisector of *AC*.

Solving (i) and (ii), we get $x = 3$, $y = -3$.

Hence, the coordinates of the circumcentre of ΔABC are (3, –3).

7. Orthocentre of a Triangle The orthocentre of a triangle is the point of intersection of the perpendiculars drawn from the vertices to the opposite sides of the triangle.

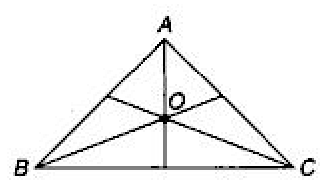


Fig. 19.8

Illustration

Question 7: Find the orthocentre of the triangle whose sides have the equations $y = 15$, $3x = 4y$, and $5x + 12y = 0$.

7. PQR is an isosceles triangle. If the coordinates of the base are Q (1, 3) and R (–2, 7), then the coordinates of the vertex P can be

(a) $\left(4, \frac{7}{2}\right)$ (b) (2, 5)

(c) $\left(\frac{5}{6}, 6\right)$ (d) $\left(\frac{1}{3}, 2\right)$

8. The extremities of a diagonal of a parallelogram are the points (3, – 4) and (– 6, 5). If the third vertex is the point (–2, 1), the coordinate of the fourth vertex is

(a) (1, 0) (b) (–1, 0)
(c) (–1, 1) (d) (1, –1)

9. If the points (a, 0), (0, b) and (1, 1) are collinear then which of the following is true?

(a) $\frac{1}{a} + \frac{1}{b} = 2$ (b) $\frac{1}{a} - \frac{1}{b} = 1$

(c) $\frac{1}{a} - \frac{1}{b} = 2$ (d) $\frac{1}{a} + \frac{1}{b} = 1$

10. If P and Q are two points on the line $3x + 4y = -15$, such that $OP = OQ = 9$ units, the area of the triangle POQ will be

(a) $18\sqrt{2}$ sq units (b) $3\sqrt{2}$ sq units
(c) $6\sqrt{2}$ sq units (d) $15\sqrt{2}$ sq units

11. If the coordinates of the points A, B, C and D are (6, 3), (–3, –5), (4, –2) and (a, $3a$) respectively and if the ratio of the area of triangles ABC and DBC is 2 : 1, then the value of a is

(a) $\frac{-9}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-23}{36}$ (d) $\frac{23}{18}$

12. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. What will be the length of the intercept cut by the side BC on the y-axis?

(a) $\frac{9}{5}$ (b) 8
(c) 1.5 (d) No unique solution

13. A line is represented by the equation $4x + 5y = 6$ in the coordinate system with the origin (0, 0). You are required to find the equation of the straight line perpendicular to this line that passes through the point (1, –2) [which is in the coordinate system where origin is at (–2, –2)].

(a) $5x - 4y = 11$ (b) $5x - 4y = 13$
(c) $5x - 4y = -3$ (d) $5x - 4y = 7$

14. P (3, 1), Q (6, 5) and R (x, y) are three points such that the angle PRQ is a right angle and the area of ΔPRQ is 7. The number of such points R that are possible is

(a) 1 (b) 2 (c) 3 (d) 4

15. Two sides of a square lie on the lines $x + y = 1$ and $x + y + 2 = 0$. What is its area?

(a) $\frac{11}{2}$ (b) $\frac{9}{2}$ (c) 5 (d) 4

16. Find the value of k if the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ is perpendicular to the line $7x + 5y = 4$.

(a) $\frac{-33}{37}$ (b) $\frac{-29}{37}$

(c) $\frac{19}{37}$ (d) None of these

17. If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which of the following is true?

(a) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (d) None of these

18. How many points on $x + y = 4$ are there that lie at a unit distance from the line $4x + 3y = 10$?

(a) 1 (b) 2
(c) 3 (d) None of these

19. What will be the area of the rhombus $ax \pm by \pm c = 0$?

(a) $\frac{3c^2}{ab}$ (b) $\frac{4c^2}{ab}$ (c) $\frac{2c^2}{ab}$ (d) $\frac{c^2}{ab}$

20. The coordinates of the mid-points of the sides of a triangle are (4, 2), (3, 3) and (2, 2). What will be the coordinates of the centroid of the triangle?

(a) $\left(3, \frac{7}{3}\right)$ (b) $\left(-3, \frac{-7}{3}\right)$

(c) $\left(3, \frac{-7}{3}\right)$ (d) $\left(-3, \frac{7}{3}\right)$

Hints and Solutions II

1. Use the area of a triangle formula for the two parts of the quadrilateral separately and then add them.